



Calhoun: The NPS Institutional Archive

Theses and Dissertations

Thesis Collection

1993-12

Trajectory planning for space manipulators

Zurowski, Mary M.

Monterey, California. Naval Postgraduate School

<http://hdl.handle.net/10945/39767>



Calhoun is a project of the Dudley Knox Library at NPS, furthering the precepts and goals of open government and government transparency. All information contained herein has been approved for release by the NPS Public Affairs Officer.

Dudley Knox Library / Naval Postgraduate School
411 Dyer Road / 1 University Circle
Monterey, California USA 93943

<http://www.nps.edu/library>

2

NAVAL POSTGRADUATE SCHOOL
Monterey, California

AD-A277 229



DTIC
ELECTE
MAR 25 1994
S E D

THESIS

TRAJECTORY PLANNING FOR SPACE MANIPULATORS

by

Mary M. Zurowski

December 1993

Thesis Advisor:

Ranjan Mukherjee

Approved for public release; distribution is unlimited.

7488

94-09103



DTIC QUALITY INSPECTED 1

94 3 22 028

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington DC 20503.				
1. AGENCY USE ONLY		2. REPORT DATE 16 December 1993		3. REPORT TYPE AND DATES COVERED Master's Thesis
4. TITLE AND SUBTITLE: TRAJECTORY PLANNING FOR SPACE MANIPULATORS			5. FUNDING NUMBERS	
6. AUTHOR(S) <i>Zurowski, Mary M.</i>				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000			8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)			10. SPONSORING/MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.				
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.			12b. DISTRIBUTION CODE *A	
13. ABSTRACT The angular momentum of a free-flying multibody system in space is a conserved quantity. This conservation law acts as a nonholonomic constraint and manifests itself when cyclic motion of the articulated joints of an on board manipulator produces a net change in the orientation of the whole system. This poses two important and coupled problems: (a) the motion planning problem of the manipulator for attitude reorientation of the space structure using internal motion of the joints, and (b) planning the manipulator joint trajectories that produce repeatable motion of all the configuration variables. We have adopted a surface integral approach to come up with algorithms for these nonholonomic motion planning problems.				
14. SUBJECT TERMS (1) Satellite Attitude Control, (2) Non-holonomic Motion Planning, (3) Linked Multi-bodied Space Structures			15. NUMBER OF PAGES 74	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT UL	

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI STD. 239-18

Approved for public release; distribution is unlimited.

--
Trajectory Planning for Space Manipulators

by

Mary M. Zurowski
Lieutenant, United States Navy
B.A.Sc., University of Toronto, 1983

Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
December 1993

Author:

Mary M. Zurowski

Approved by:

Ranjan Mukherjee, Thesis Advisor

Matthew D. Kelleher, Chairman
Department of Mechanical Engineering

ABSTRACT

The angular momentum of a free-flying multibody system in space is a conserved quantity. This conservation law acts as a nonholonomic constraint and manifests itself when cyclic motion of the articulated joints of an on board manipulator produces a net change in the orientation of the whole system. This poses two important and coupled problems: (a) the motion planning problem of the manipulator for attitude reorientation of the space structure using internal motion of the joints, and (b) planning the manipulator joint trajectories that produce repeatable motion of all the configuration variables. We have adopted a surface integral approach to come up with algorithms for these nonholonomic motion planning problems.

Accession For	
NTIS	CRA&I <input checked="" type="checkbox"/>
DTIC	TAB <input type="checkbox"/>
Unannounced <input type="checkbox"/>	
Justification _____	
By _____	
Distribution /	
Availability Codes	
Dist	Avail and/or Special
A-1	

TABLE OF CONTENTS

I. INTRODUCTION	1
II. MATHEMATICAL PRELIMINARIES	10
A. RELEVANT THEOREMS	10
1. Theorem 1: Exactness [Ref. 5]	10
2. Theorem 2: Exactness and Independence of Path [Ref. 14]	11
3. Theorem 3: Stoke's Theorem [Ref. 2]	11
B. MANIFESTATION OF NONHOLONOMY	12
C. THE SURFACE INTEGRAL APPROACH: ATTITUDE CONTROL	13
D. THE NECESSARY CONDITION FOR REPEATABILITY . . .	15
III. THE SURFACE INTEGRAL APPROACH: ATTITUDE CONTROL .	19
A. INTRODUCTION AND NOMENCLATURE	19
B. ALGORITHM FOR MOTION PLANNING	20
1. Y - Class Motion	22
2. Z - Class Motion	26
3. X - Class Motion	27
C. SYNTHESIS OF MANIPULATOR MOTION FOR REORIENTATION	28
D. SIMULATION RESULTS	32

IV. PLANNING REPEATABLE PATHS FOR PLANAR SPACE ROBOTS	45
A. INTRODUCTION	45
B. NECESSARY CONDITION FOR REPEATABILITY	46
C. SIMULATION RESULTS	51
V. CONCLUSIONS AND RECOMMENDATIONS	61
A. CONCLUSIONS	61
B. RECOMMENDATIONS	62
APPENDIX A	63
LIST OF REFERENCES	64
INITIAL DISTRIBUTION LIST	67

ACKNOWLEDGEMENTS

..

To Our Lord and Savior Jesus Christ, who holds us in the palm of His hand.

I would like to acknowledge my sincere appreciation to: Dr. Ranjan Mukherjee, my thesis advisor, for his expert guidance and instruction during the course of this thesis; and, to David Marco, of Naval Engineering for his computer programming assistance.

Heartfelt thanks goes to four friends, Pam Davis, LCDR Craig Bateman, USN, LT Mary Murphy, USN, and LT Mary Logsdon, USN, for their help with the typing, and editing of this document. Further, my deepest gratitude goes to LCDR Craig Bateman for his tireless tutoring and patience with this thesis and all my other endeavors at the Postgraduate School.

I. INTRODUCTION

There is a growing interest in the area of attitude control and motion planning of multi-body systems in space. These structures in space are expected to have attached articulated joint manipulator arms on board. A problem arises for these structures in that the movement of the manipulator arm will cause a displacement for the whole structure. This displacement is a result of the dynamic coupling between the arm and the structure. Multi-body systems in space, in the absence of external forces, conserve the angular momentum of the system. This conservation acts as a nonholonomic constraint on the motion of the system. For structures with attached manipulator arms, this conservation law manifests itself when cyclic motion of the manipulator joints produce a net change in the orientation, i.e., a drift in the orientation, of the whole system. Changes in system orientation can also arise from other causes such as: (1) differential gravitational forces; (2) solar radiation effects; (3) dynamic interactions between a space station and on board robots or a docking shuttle craft; and, (4) the operation of booster rockets used for orbit maintenance [Ref. 20].

An extensive literature survey will lead one to believe that the best way for attitude control would be to use momentum exchange devices with control momentum gyroscopes as the most desirable devices for attitude control.

Though the most desirable devices, gyroscopes have some disadvantages which are: (1) they require a steady power source to overcome the dissipated energy of the friction in the bearings; (2) susceptibility to mechanical failure as a result of its constant motion; and, (3) a significant added weight effect. Other devices such as booster rockets and gas jets have the disadvantage of: (1) requiring onboard fuel storage which adds a considerable weight effect; and, (2) fuel sources that once expended are non-replenishable without a considerable monetary expense.

If manipulators can effectively be used to reorient a space structure, they can serve as a reliable back-up means of attitude control in the event of a power interruption or mechanical failure of the gyroscope. In the case of a small satellite with an attached manipulator where the added mass of a gyroscope or booster rocket fuel is undesirable, the manipulator can serve the dual purpose of attitude control and automation in space.

The advantages of the manipulator are: (1) they are already aboard; (2) require much less power than momentum exchange devices; (3) are less susceptible to mechanical failure; and, (4) use of the manipulators internal controls

does not modify the total angular momentum of the system [Ref. 24].

A related issue to using a manipulator arm to reorient a space structure is that, due to the nonholonomic nature of the structure, the use of a manipulator to perform a required task may result in an undesirable change in the orientation of the structure. It thus becomes desirable to be able to predict and control the change in orientation of a freely-floating space structure. The ability to predict and control this change is the subject of this thesis.

Some of the earliest work in the study of the motion planning problems of nonholonomic systems has been done by Kane and Scher [Ref. 12], who studied the falling cat problem, and Kane, Hedrick and Yatteau [Ref. 11], who studied the astronaut maneuvering scheme.

More recently the study of the use of manipulators for reorientation of space structures has been done by Vafa and Dubowsky [Ref. 29], where cyclic motion of the joint variables were proposed to reorient the space vehicle, and Fernandes, Gurvits and Li [Ref. 4] proved the controllability of a space robot system using a three link manipulator. This work motivated Nakamura and Mukherjee [Ref. 22], who proposed a bi-directional approach to the motion planning of free-flying space robots to control both the space vehicle orientation and the manipulator joints by actuating only the manipulator joints. Conversely, Yamada and Yoshikawa [Ref. 32] prescribed

an arm trajectory and then found the optimal trajectory that yielded the desired attitude change with the minimum arm movement. Walsh and Sastry [Ref. 31] provided kinematic algorithms for reorienting some systems of linked rigid bodies floating in space.

Other studies on the control, stabilization, repeatability, drift and motion planning for reorientation of linked multi-bodied structures in space can be found in the reference section.

This thesis follows on with work done by Mukherjee and Anderson [Ref. 19], wherein they proposed a method of the surface integral approach for planning the motion of a two dimensional nonholonomic system.

In this thesis we present two concepts. The first is an algorithm for the motion planning of a space manipulator to achieve attitude control of a freely-floating three dimensional space structure. Generally stated the algorithm provides a means for calculating the coordinate trajectories required to drive a nonholonomic system from one point in its configuration space to some other desired point. The algorithm invokes the use of Stokes' Theorem and, therefore, takes a surface integral approach to the problem as is done in [Ref. 20].

Secondly, we present a means of determining the manipulator motion required for the nonholonomic freely floating space structure to behave in a holonomic manner

globally, which we call "pseudo-holonomic behavior" [Ref. 21]. The method determines if "holonomic loops" [Ref. 21] do exist, where the system exhibits holonomic behavior globally for particular paths in the configuration space of the nonholonomic system. The planar space robot is the system studied and its configuration space is the joint space of the manipulator. If a "holonomic loop" does exist, we present an algorithm for finding that loop within the configuration space.

This thesis is organized as follows:

Chapter II presents some mathematical preliminaries necessary for understanding the behavior of nonholonomic systems.

Chapter III studies the freely floating three dimensional space structure with an attached three link manipulator as shown in Figure 1.1. The problem to be solved is to change the orientation of the structure from one configuration to another configuration by moving the manipulator arm joints along pre-planned paths. An example of the initial and final orientations of the structure are as shown in Figure 1.2. Chapter III provides an algorithm for planning the path of the manipulator joints necessary to achieve a desired change in orientation of the structure. Once the path is planned a simulation is conducted to illustrate that manipulators can indeed reorient a space structure.

Chapter IV studies the freely-floating planar space robot with an attached two link manipulator arm as shown in Figure 1.3. The problem to be solved is how to plan the path of the manipulator arm joints such that the space robot will not reorient itself in space. This amounts to finding the path in space for the manipulator arm where the planar space robot exhibits holonomic behavior globally. Chapter IV provides an algorithm for planning the path of the manipulator joints that will allow the planar space robot to regain its original orientation after the manipulator motion is complete. Once the path is planned a simulation is conducted, illustrating that repeatable motion is possible for nonholonomic systems.

Chapter V presents conclusions and recommendations.

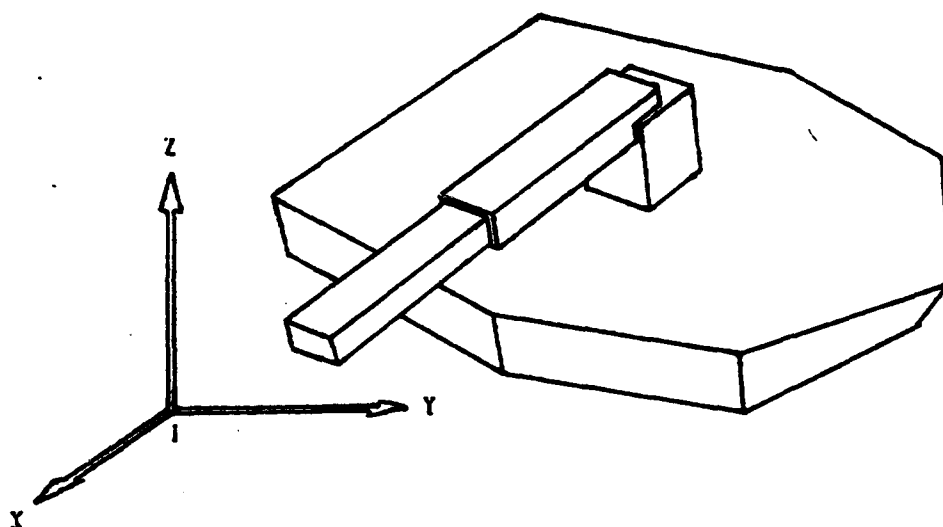
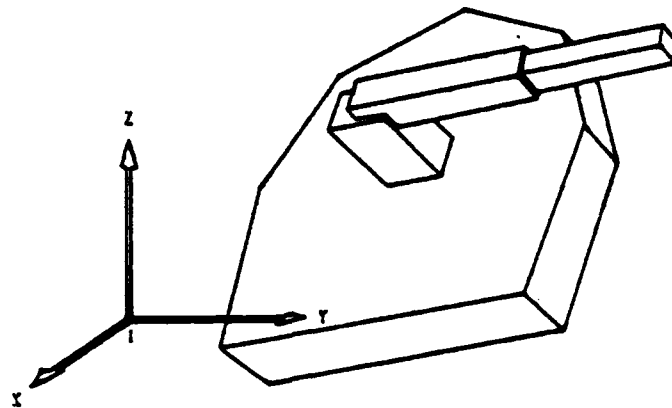
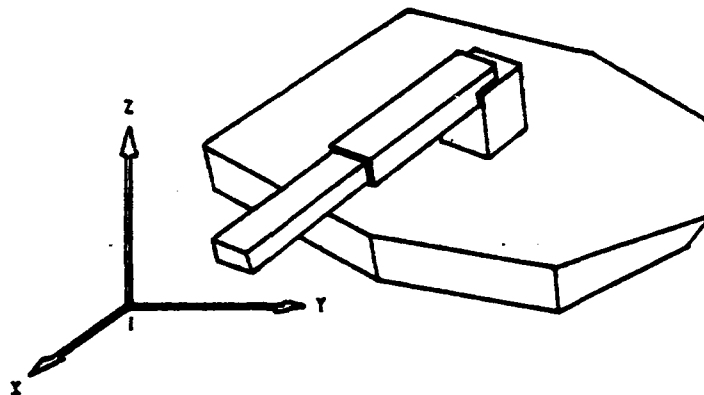


Figure 1.1 A Freely-Floating Space Structure with a 3 Link Manipulator.



(a)



(b)

Figure 1.2

The (a) Initial, and (b) Final Orientation of a Structure in Space.

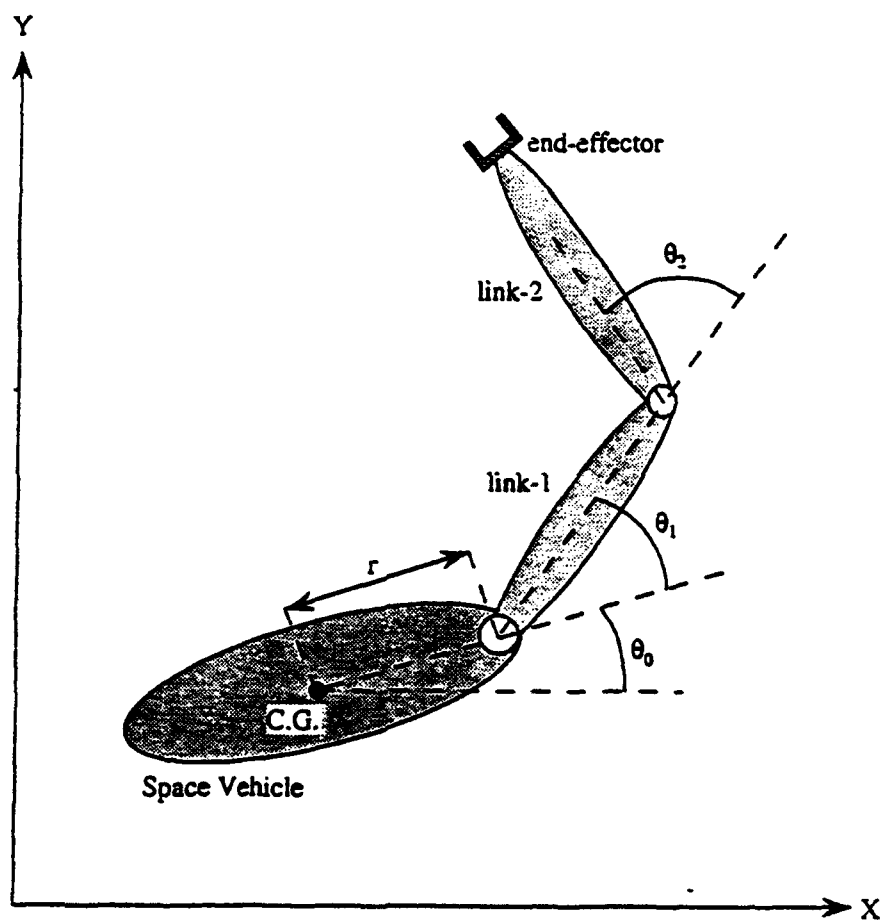


Figure 1.3 A Planar Space Robot With Two Links.

II. MATHEMATICAL PRELIMINARIES

A. RELEVANT THEOREMS

A review of a few mathematical theorems is necessary to understand how nonholonomic systems behave and the problem solutions which we propose. The first theorem concerns the exactness and integrability of a differential equation. The second concerns the path independence of line integrals. Finally, the third is Stoke's Theorem, which transforms line integrals into surface integrals.

1. Theorem 1: Exactness [Ref. 5]

A differential expression of the form

$$M(x, y, z) dx + N(x, y, z) dy + P(x, y, z) dz \quad (2-1)$$

is exact on a domain D in space if,

$$Mdx + Ndy + Pd z = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz \quad (2-2)$$

for some (scalar) function f throughout D. The differential form Equation (2-1) is exact, if and only if;

$$\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z}, \quad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}, \quad \text{and} \quad \frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}. \quad (2-3)$$

It is a well established fact that a differential expression is integrable, if it is exact or can be made exact, by multiplying it by an integrating factor. In other words, exactness implies integrability. Therefore, it follows that non-integrability implies non-exactness. Exactness is also a necessary and sufficient condition for path independence of line integrals. This is stated formally next.

2. Theorem 2: Exactness and Independence of Path
[Ref. 14]

Let $f(x,y,z)$, $g(x,y,z)$ and $h(x,y,z)$ be continuous functions in a domain D in space, then the line integral is

$$\oint_C (f dx + g dy + h dz) \quad (2-4)$$

is independent of the path C in D , if and only if, the differential form under the integral sign in Equation (2-4) is exact in D . Additionally the line integral is independent of path in D if and only if it is zero on every simple closed path C in D .

A line integral over a closed path C can be converted the into a surface integral utilizing the well known Stoke's Theorem [Ref. 2].

3. Theorem 3: Stoke's Theorem [Ref. 2]

If D is a k -dimensional space and ω is a $(k-1)$ differential form on D , then from Stoke's theorem we have

$$\int_{\partial D} \omega = \int_D d\omega \quad (2-5)$$

where, ∂D is the path of the line integration and is the boundary of the domain D , and $d\omega$ is a differential k form, obtained by the exterior differentiation of ω .

B. MANIFESTATION OF NONHOLONOMY

The importance of theorem 2 lies in the fact that nonholonomic systems are governed by non-integrable and, hence non-exact differential constraint equations. Nonholonomic systems are, therefore, path dependent. To illustrate path dependency consider the following equation:

$$dp = v_1 dx + v_2 dy + v_3 dz \quad (2-6)$$

Where: (1) p is the dependent variable of a nonholonomic system; (2) x , y and z are the independent variables; and, (3) v_1 , v_2 and v_3 are continuous functions of x , y and z . Since the system is non-integrable the differential form,

$$v_1 dx + v_2 dy + v_3 dz$$

is not exact. Therefore, by theorem 2 the change in p is path dependent. Hence, it is possible to change the coordinates of the dependent variable p , by using closed trajectories of the independent variables, as shown in Figure (2.1). The independent variables, which trace the closed path C , start at point two, move around the path and return to point five,

coincident with point two. The independent variables have returned to their original value, whereas the dependent variable p has taken on a new value, $p + \Delta p$. Path dependency is a characteristic of nonholonomic systems, which we will use to reorient a structure in space using angular momentum preserving controls; this is considered next.

C. THE SURFACE INTEGRAL APPROACH: ATTITUDE CONTROL

In Chapter I, we have seen that freely floating structures in space are nonholonomic systems. The nonholonomy arises from the fact that the conservation of angular momentum yields non-integrable constraints of motion. It is this non-integrability that permits the reorientation of the structure while maintaining a zero value of angular momentum.

In the case of an articulated space structure, the nonholonomic constraint equations relate the rate of change of the dependent variables, the structure orientation, to the rate of change of the independent variables, the angles of the articulated arms. To achieve the desired change in the orientation of the structure, we need only find the correct path in the configuration space of the independent variables, the joint angles, to yield the desired change in the dependent variables, the orientation of the structure. We can find this path by methodically utilizing the surface integral approach to solve for the closed path.

The nonholonomic motion constraint equation for the articulated structure can be expressed in a differential form. Integration of this differential form, to obtain the change in the dependent variable, amounts to solving the line integral of the equation in the space of the dependent variables. The surface integral approach utilizes Stoke's theorem to convert the line integral into a surface integral. This approach simplifies the mathematics and allows us to appropriately choose a surface area, which will yield the desired change in the dependent variable. Once the surface area is chosen, the path enclosing the surface area can be found by setting the limits of integration. The change in the dependent variables can now be found as a function of the limits of integration. By choosing the limits of integration, we have the ability to satisfy additional constraints, such as the limits on the values of the independent variables.

To illustrate this, consider an arbitrary space structure as previously shown in Figure 1.1 with an attached manipulator arm. Suppose that the manipulator has constraints on its motion, such as joint limits or work space limitations. Now suppose we wish to reorient the structure by an amount ϕ where $\phi = nk$, where $n = 1, 2, \dots$, and k is the change in orientation, as the manipulator traverses a path c . By appropriately setting the limits of integration, we can choose our surface such that we reorient the structure by traversing a path C one time or traversing a smaller path c , n times. The manipulator

motion can then be planned amidst additional constraints, such as manipulator joint limits and environmental work space limitations. The surface integral approach will be presented in detail in Chapter III.

D. THE NECESSARY CONDITION FOR REPEATABILITY

The property of repeatability of a system is that, when the independent variables move along closed trajectories, the dependent variables also move along closed trajectories. Repeatability is ensured if the differential constraints of motion of a system are integrable and, hence, are path independent. Naturally holonomic systems exhibit this property.

The purpose of Chapter IV is to demonstrate that: (1) integrability of the differential constraint is only a sufficient condition for repeatability, but it is by no means a necessary condition; and, (2) that a necessary condition for the repeatable motion of a nonholonomic system is as follows:

Consider a two dimensional path dependent system where θ_0 is the dependent variable and θ_1 and θ_2 are the independent variables. Suppose also that the dependency of θ_0 on θ_1 and θ_2 is explicitly given by the following equation:

$$d\theta_0 = g_1 d\theta_1 + g_2 d\theta_2 \quad (2-7)$$

where g_1 and g_2 are functions of θ_1 and θ_2 . The change in the dependent variable is given by the line integral.

$$\begin{aligned}\int d\theta_0 &= \int [g_1 d\theta_1 + g_2 d\theta_2] \\ &= \int_D \alpha \left[\frac{\partial g_2}{\partial \theta_1} - \frac{\partial g_1}{\partial \theta_2} \right] d\theta_1 \wedge d\theta_2 \\ &= \int_D \left[\frac{\partial g_2}{\partial \theta_1} - \frac{\partial g_1}{\partial \theta_2} \right] (d\theta_1 \wedge d\theta_2) \cdot \alpha\end{aligned}\tag{2-8}$$

applying Stokes' theorem yields,

$$\int_{\partial D} d\theta_0 = \pm \int_D \left[\frac{\partial g_2}{\partial \theta_1} - \frac{\partial g_1}{\partial \theta_2} \right] d\theta_1 d\theta_2\tag{2-9}$$

where " \wedge " denotes the exterior product, " \cdot " denotes the dot product and α , the orientation of D has the same orientation as $dx_1 \wedge dx_2$ when the direction along the path is counterclockwise, otherwise α has the same orientation as $dx_2 \wedge dx_1$. For a nonholonomic system,

$$\frac{\partial g_2}{\partial \theta_1} \neq \frac{\partial g_1}{\partial \theta_2}$$

hence, we define

$$\left(\frac{\partial g_2}{\partial \theta_1} - \frac{\partial g_1}{\partial \theta_2} \right) \triangleq F(\theta_1, \theta_2)\tag{2-10}$$

substituting into Equation (2-9) we get

$$\begin{aligned}
 \int_{\partial D} d\theta_0 &= \int_D F(\theta_1, \theta_2) d\theta_1 d\theta_2 \\
 &= F(\theta_1^*, \theta_2^*) \int_D d\theta_1 d\theta_2 \\
 &= F(\theta_1^*, \theta_2^*) \pi(D) , \quad (\theta_1^*, \theta_2^*) \in D
 \end{aligned}
 \tag{2-11}$$

Equation (2-10) was obtained by the application of the mean value theorem of integral calculus. The function F can be shown to be continuous in the entire domain D and, hence, the mean value theorem applies. θ_1^* and θ_2^* denote some point within the domain D ; and, $\pi(D)$ is the measure of the domain D ; in this case it is simply equal to the area enclosed within the closed curve ∂D . $F(\theta_1^*, \theta_2^*)$ can also be interpreted as the mean value of the function F , defined by Equation (2-10), taken over the domain D . If this mean value happens to be zero, then we would have no net change in the dependent variable as the independent variables move along closed paths and return to their original value. Hence, we have a nonholonomic system that exhibits pseudo holonomic behavior. We will apply this concept to a planar space robot in Chapter IV.

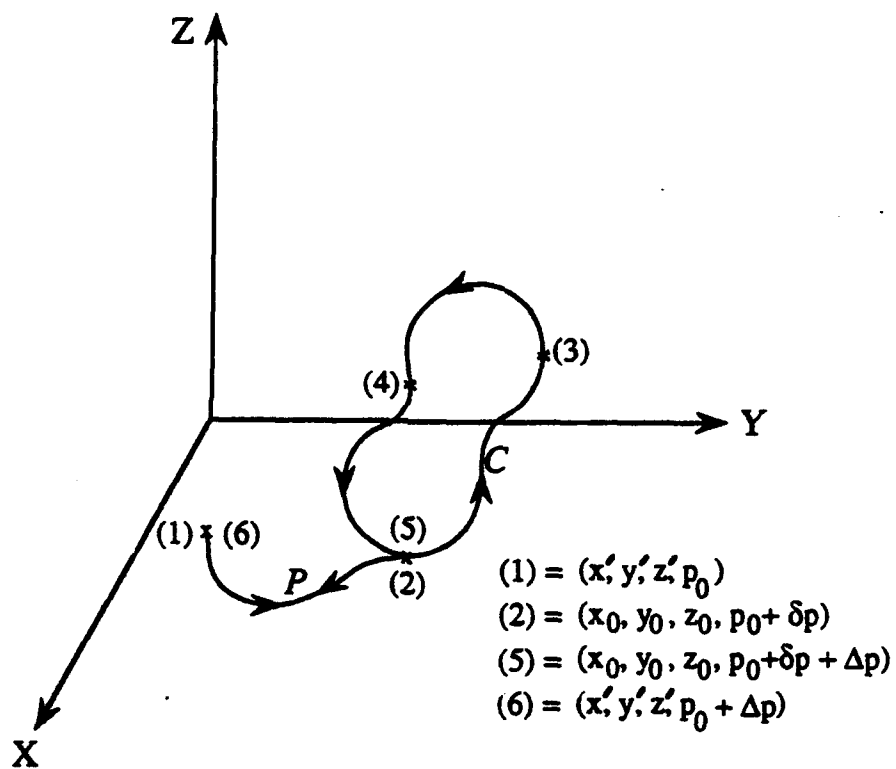


Figure 2.1: The Closed Trajectory C in the Independent Variables X, Y, and Z Produces a Change in the Dependent Variable P by an Amount Δp .

III: THE SURFACE INTEGRAL APPROACH: ATTITUDE CONTROL

A. INTRODUCTION AND NOMENCLATURE

In Chapter I, we saw that for a freely-floating space structure with an attached manipulator arm that if the manipulator can effectively be used to reorient the structure then manipulators can serve: (1) as a reliable back-up to gyroscopes; and, (2) the dual purpose of attitude control and automation in space.

In this chapter, we develop the algorithm for attitude control of a space structure using a three link manipulator and present the results of the simulations. We assume the robot to have a PUMA type structure as shown in Figure 3.1(a), and the reference frames are according to the Denavit-Hartenburg [Ref. 3] convention. Figure 3.1(b) shows the kinematic structure of the manipulator. For mathematical simplicity: (1) the center of mass of the system was chosen to coincide with the geometric center of the body; and, (2) the inertial and body fixed axes were chosen to be coincident at point S.

The following nomenclature is used throughout the development:

frame I Inertia frame.

frame S	Frame fixed at the center of mass of the space structure and directed along the principal axes of the structure.
..	
frame K	The k -th kink frame of the manipulator according to the Denavit-Hartenburg [3] convention. $k = 0$ denotes the manipulator base frame.
m_k	Mass of the k -th body for $k = 1, 2, 3...$ the mass of the space structure is m_s (kg).
${}^k I_k \in \mathbb{R}^{3 \times 3}$	Inertial matrix of the k -th body about the principal axes located at the center of mass, and expressed in the k -th link frame. The inertia matrix of the space structure is denoted by ${}^S I_s$, (kgm^2).
I_{kij}, I_{sij}	(i, j) -th element of ${}^k I_k, {}^S I_s$ (kgm^2).
x_0, y_0, z_0	Position of the center of mass of the space structure in the inertia frame (m).
ϕ_1, ϕ_2, ϕ_3	y-x-y Euler angles, describing the orientation of the space structure with respect to the inertia frame.
$\beta_0, \beta_1, \beta_2, \beta_3$	Euler parameters [Ref. 10].
$R[\bullet, *] \in \mathbb{R}^{3 \times 3}$	Orthogonal rotation matrix corresponding to a rotation of the (\bullet) axis fixed on the space structure by an angle ($*$).
$(\theta_1, \theta_2, \theta_3)$	Joint configuration of the three link manipulator.

B. ALGORITHM FOR MOTION PLANNING

The angles and parameters of interest in the development of the algorithm for motion planning are: (1) Euler angles, describing the orientation of the space structure; (2) Euler parameters; and, (3) the joint angles of the manipulator. The "home" and "intermediate" configurations of the manipulator are as shown in Figure 3.2. The "home" configuration is

defined as θ_1 , θ_2 , and θ_3 equal to zero degrees where θ_1 , θ_2 , and θ_3 are the joint angles of the first, second, and third links, respectively. The "intermediate" configuration is defined as θ_1 equal to ninety degrees and θ_2 and θ_3 equal to zero degrees.

The reorientation of the space structure will be achieved through rotations of the structure about the body fixed x, y, and z axes. Our goal is to change the orientation of the space structure from an initial set of Euler angles ϕ_{1i} , ϕ_{2i} , ϕ_{3i} to a desired set of values ϕ_{1f} , ϕ_{2f} , ϕ_{3f} without any change in the system configuration. In other words, we would like the manipulator to have the same joint configuration (θ_1 , θ_2 , θ_3), say the "home" configuration, when the orientation of the structure is $(\phi_1, \phi_2, \phi_3) = (\phi_{1i}, \phi_{2i}, \phi_{3i})$ or $(\phi_{1f}, \phi_{2f}, \phi_{3f})$. The initial and final configurations are given in Figure 3.3.

Three classes of motion are defined as follows:

(1) Y - Class motion

The purpose of the class Y motion is to change the orientation of the space structure about its' Y_S axis using the manipulator. The manipulator will be at the "home" configuration at the beginning and end of this motion. Furthermore, during this motion the first joint of the manipulator will be kept fixed at $\theta_1 = 0.0$ radians. The motion of the manipulator will, therefore, remain confined to the X_S - Z_S plane, and the problem will reduce to a planar problem.

(2) Z - Class motion

The purpose of the class Z motion is only to reconfigure the manipulator. It will be used to bring the manipulator to the "intermediate" configuration from the "home" configuration, and vice versa. The reconfiguration will be achieved by using only the first joint of the

manipulator. The second and third joints of the manipulator will be held fixed at $(\theta_2, \theta_3) = (0,0)$ during this motion. The motion of the manipulator will, therefore, remain confined to the X_S - Y_S plane. Note that the Z-class motion is a holonomic motion because only one manipulator joint is involved in this motion.

(3) X - Class motion

The purpose of the class X motion is to change the orientation of the space structure about its' X_S axis using the manipulator. The manipulator will be at the "intermediate" configuration at the beginning and end of this motion. Furthermore, during this motion the first joint of the manipulator will be kept fixed at $\theta_1 = \pi$ radians. The motion of the manipulator will, therefore, remain confined to the Y_S - Z_S plane.

To reorient the structure there are twelve possible combinations of rotations about the body fixed axes. The scheme chosen for this problem was to sequentially rotate the structure about its' y-x-y axes.

1. Y - Class Motion

The change in orientation, rotation of the structure about its' Y_S axis is given by the nonholonomic angular momentum constraint equation as follows:

$$\dot{\phi} = \frac{1}{\Delta} (a\dot{\theta}_2 + b\dot{\theta}_3) , \quad (3-1a)$$

$$a \triangleq C_1 \sin \theta_2 + C_2 \cos \theta_3 + C_3 \sin(\theta_2 + \theta_3) + s_1 ,$$

$$b \triangleq C_2 \cos \theta_3 + C_3 \sin(\theta_2 + \theta_3) + s_2 ,$$

$$\Delta \triangleq 2C_1 \sin \theta_2 + C_2 \cos \theta_3 + 2C_3 \sin(\theta_2 + \theta_3) - s_3 ,$$

where the constants C_1 , C_2 , C_3 , s_1 , s_2 and s_3 are defined as,

$$\begin{aligned}
 C_1 &\triangleq l_2 (0.5m_2 + m_3) [m_s(r + l_1) + 0.5m_1 l_1] , \\
 C_2 &\triangleq m_3 l_2 l_3 (m_s + m_1 + 0.5m_2) , \\
 C_3 &\triangleq 0.5m_3 l_3 [m_s(r + l_1) + 0.5m_1 l_1] , \\
 s_1 &\triangleq m_t [I_{233} + (0.25m_2 + m_3) l_2^2] - (0.5m_2 + m_3)^2 l_2^2 + s_2 , \\
 s_2 &\triangleq m_t [I_{333} + 0.25m_3 l_3^2] - 0.25m_3^2 l_3^2 , \\
 s_3 &\triangleq -m_t [I_{s22} + I_{133} + (m_2 + m_3) (r + l_1)^2 + m_1 (r + 0.5l_1)^2] \\
 &\quad - [m_1 (r + 0.5l_1) + (m_2 + m_3) (r + l_1)]^2 - s_1 ,
 \end{aligned} \tag{3-1b}$$

and where, $m_t = (m_s + m_1 + m_2 + m_3)$, and r , l_1 , l_2 and l_3 are defined in Figure 3.1(b). Equation (3-1a) gives the angular velocity of the structure about its' Y_s axis as a function of the joint configuration and joint velocities.

If the second and third joints of the manipulator move along a closed path C in the θ_2 - θ_3 plane, then the net change of orientation of the spacecraft about its' Y_s axis is given by;

$$\begin{aligned}
 \phi_Y &= \int \alpha \phi = \oint_C \frac{1}{\Delta} (a d\theta_2 + b d\theta_3) \\
 &= \iint_S \left[\frac{\partial}{\partial \theta_2} \left(\frac{b}{\Delta} \right) - \frac{\partial}{\partial \theta_3} \left(\frac{a}{\Delta} \right) \right] d\theta_2 d\theta_3 ,
 \end{aligned} \tag{3-2}$$

where, S is the surface area enclosed within the closed curve C . Choosing the surface area S to be rectangular, to simplify the integration, and specifying three of the four limits of integration we can solve for the change in orientation as a function of the fourth limit of integration. The fourth, and unknown, limit of integration was chosen as θ_{2u} . After integrating, the final expression for the change in orientation, ϕ_y , as a function of θ_{2u} , Equation (3-3) was obtained.

$$\begin{aligned}
 \phi_y = & -\frac{C_2 C_3 \cos \theta_{2u}}{v_5} \ln \left(\frac{s_3 - 2C_1 \sin \theta_{2u} - 2C_3 \cos \theta_{2u}}{s_3 - C_2 - 2(C_1 + C_3) \sin \theta_{2u}} \right) - \frac{\pi v_4}{2v_5} \\
 & + \frac{2[s_2 v_5 + (s_3 - 2C_1 \sin \theta_{2u}) v_4]}{v_5 v_6} x \\
 & \quad \arctan \left(\frac{s_3 + C_2 - 2(C_1 - C_3) \sin \theta_{2u} - 2C_3 \cos \theta_{2u}}{v_6} \right) \\
 & - \frac{2[s_2 v_5 + (s_3 - 2C_1 \sin \theta_{2u}) v_4]}{v_5 v_6} \arctan \left(\frac{-2C_3 \cos \theta_{2u}}{v_6} \right) \\
 & + \frac{v_1}{v_2} \left[\frac{\pi}{2} + \ln \left(\frac{2C_3 - s_3}{C_2 - s_3} \right) \right] \\
 & + \frac{2(s_2 v_2 + s_3 v_1)}{v_2 v_3} \left[\arctan \left(\frac{2C_3 - C_2 - s_3}{v_3} \right) - \arctan \left(\frac{2C_3}{v_3} \right) \right] \\
 & + \frac{s_3 + 2s_1}{v_7} \left[\arctan \left(\frac{2C_1 - (s_3 + 2C_3) \tan(\theta_{2u}/2)}{v_7} \right) - \arctan \left(\frac{2C_1}{v_7} \right) \right] \\
 & - \frac{s_3 + 2s_1 + C_2}{v_8} \arctan \left(\frac{2(C_1 + C_3) + (C_2 - s_3) \tan(\theta_{2u}/2)}{v_8} \right) \\
 & - \frac{s_3 + 2s_1 + C_2}{v_8} \arctan \left(\frac{2(C_1 + C_3)}{v_8} \right)
 \end{aligned} \tag{3-3a}$$

with

$$\begin{aligned}
 v_1^2 &\triangleq C_2^2 + 2C_3^2, \\
 v_2^2 &\triangleq C_2^2 + 4C_3^2, \\
 v_3^2 &\triangleq s_3^2 - C_2^2 - 4C_3^2, \\
 v_4 &\triangleq v_1 + 2C_2C_3\sin\theta_{2u}, \\
 v_6^2 &\triangleq (s_3 - 2C_1\sin\theta_{2u})^2 - (C_2 + 2C_3\sin\theta_{2u})^2 - 4C_3^2\cos^2\theta_{2u}, \\
 v_7^2 &\triangleq s_3^2 - 4C_3^2 - 4C_1^2, \\
 v_8^2 &\triangleq (C_2 - s_3)^2 - 4(C_1 + C_3)^2.
 \end{aligned} \tag{3-3b}$$

The relationship of ϕ_y expressed as a continuous function of θ_{2u} , Equation (3-3), for $0 \leq \theta_{2u} \leq \pi$ is shown in Figure 3.4.

Since the joints of the manipulator will have physical limits, the maximum absolute value of ϕ_y will be limited by the maximum value of θ_{2u} . Referring to Figure 3.4, if we impose the joint limit of $\theta_{2u} \leq (3\pi/4)$ radians, then the maximum change in ϕ_y will be of the order of +7.50 degrees. Note that the sign of ϕ_y can be reversed by simply traversing the closed path in the θ_2 - θ_3 plane in the opposite direction. If a change in orientation greater than $\phi_y > 7.5$ degrees is desired, then the manipulator joints will have to move along some closed path a multiple number of times. This closed path can be found as follows: Set the desired change in ϕ_y to Ω where $\Omega = \delta n$, δ denotes the change in orientation each time the manipulator traces out a path, n denotes the integer

number of times the arm must trace the path. The integer n is then obtained by maximizing

$$\delta = \Omega/n \leq 7.50$$

The value of θ_{2u} is then obtained from Equation (3-3) by setting $\theta_y = \delta$ and using a non-linear function solver to solve for the root of Equation (3-3). Visual examination of Figure 3.4 will give the range in which the root of the function will lie.

Hence, any changes in orientation of the space structure about its' Y_s axis can be achieved through a single or multiple closed looped trajectories of class Y motion.

2. Z - Class Motion

The purpose of class Z motion is solely to reconfigure the manipulator from the "home" configuration to the "intermediate" configuration and vice versa. This motion makes it possible to change from rotating the structure about its' Y_s axis to rotating the structure about its' X_s axis and vice versa. This will allow the y-x-y rotation sequence previously discussed.

The change in the orientation of the structure about its' Z_s axis is given by the holonomic Equation (3-4).

$$I_{S33}\phi_z + \left[\sum_{i=1}^3 I_{i22} + 0.25m_2l_2^2 + m_3(0.5l_3 + l_2)^2 \right] (\theta_1 + \phi_z) = 0, \quad (3-4)$$

which upon integration yields,

$$\phi_z = \pm \left(\frac{I_M}{I_{S23} + I_M} \right) \frac{\pi}{2} , \quad (3-5)$$

$$I_M \triangleq \sum_{i=1}^3 I_{i22} + 0.25m_2 l_2^2 + m_3 (0.5l_3 + l_2)^2 .$$

The change in orientation ϕ_z is negative when the manipulator moves from the "home" configuration to the "intermediate" configuration. It is positive when the manipulator moves from the "intermediate" to the "home" configuration. The absolute value of ϕ_z is, therefore, a constant whose value depends upon the inertia parameters of the system.

3. X - Class Motion

The change in orientation of the structure about its' X_s axis is computed in a similar fashion as that for the motion about the Y_s axis. All the equations developed for the Y_s class motion will hold, however, with three changes, ϕ_y is replaced by ϕ_x , the constant s_3 in Equation (3-1a) will have I_{S22} replaced with I_{S11} , and Δ in Equation (3-1a) will be replaced by $-\Delta$. ϕ_x , denoting the change in orientation of the structure about its' X_s axis can be expressed as a continuous function of θ_{2u} , for $0 \leq \theta_{2u} \leq \pi$, as shown in Figure 3.4. Again imposing the limitation of $\theta_{2u} \leq (3\pi)/4$ radians, the maximum change in ϕ_x will be of the order ≤ 6.66 degrees.

Using the same logic as for ϕ_y , we can conclude that any arbitrary change in the orientation of the space structure about its' X_s axis can be achieved.

C. SYNTHESIS OF MANIPULATOR MOTION FOR REORIENTATION

Having looked at how the orientation of the space structure changes with motion of the manipulator, the goal now is to determine the path necessary for the manipulator to traverse, so that we may achieve the desired change in the orientation of the structure. As previously discussed, in section B, we have chosen an y - x - y scheme to reorient the space structure. To do this, consider the following sequence of five rotations where the change in orientation of the structure about the X_s , Y_s , and Z_s axes are denoted by Λ_1 , Λ_2 , Λ_3 , and Λ_4 .

1. Class Y motion with $\phi_y = \Lambda_1$
2. Class Z motion with $\phi_z = \Lambda_2$
3. Class X motion with $\phi_x = \Lambda_3$
4. Class Z motion with $\phi_z = -\Lambda_2$
5. Class Y motion with $\phi_y = \Lambda_4$

Note that Λ_1 , Λ_3 , and Λ_4 are variables; whereas, Λ_2 has a constant absolute value. This is because, as previously noted, the sole purpose of the Z class motion is to reconfigure the manipulator arm.

Looking at this sequence of rotations in detail: Let the initial orientation and the desired orientation of the space

structure with respect to the inertial frame be given by the rotation matrices R_i and R_f , respectively. Then,

$$R_i \triangleq R[y, \phi_{3i}]R[x, \phi_{2i}]R[y, \phi_{1i}] \quad (3-6a)$$

$$R_f \triangleq R[y, \phi_{3f}]R[x, \phi_{2f}]R[y, \phi_{1f}] \quad (3-6b)$$

where $(\phi_{1i}, \phi_{2i}, \phi_{3i})$, and $(\phi_{1f}, \phi_{2f}, \phi_{3f})$ denote the set of Euler angles describing the initial and the desired orientation of the space structure with respect to the inertial frame. Then, the set of y - x - y Euler angles (ϕ_1, ϕ_2, ϕ_3) describing the desired orientation of the space structure with respect to the initial orientation can be solved from the following equation.

$$R[y, \phi_3]R[x, \phi_2]R[y, \phi_1] = R_f R_i^T \quad (3-7)$$

Equation (3-7) has a singularity for $\phi_2 = 0, \pm\pi$. Except for this situation, ϕ_1 , ϕ_2 and ϕ_3 can be solved uniquely from Equation (3-7). At the singular configuration(s), the orientation of the structure can be trivially depicted by one single rotation about the Y_s axis of magnitude $(\phi_1 + \phi_3)$ for $\phi_2 = 0$, and of magnitude $(\phi_1 - \phi_3)$ for $\phi_2 = \pm \pi$.

Consider the sequence of rotation of the manipulator:

1. Class Y motion with $\phi_y = \Lambda_1$. The change in the orientation of the structure can be represented by $R[y, \Lambda_1]$. At the end of this motion the manipulator returns to the "home" configuration.
2. Class Z motion with $\phi_z = \Lambda_2$. Λ_2 is obtained from Equation (3-5) as:

$$\Lambda_2 = -\left(\frac{I_M}{I_M + I_{S33}}\right) \frac{\pi}{2} \quad (3-8)$$

The change in the orientation of the structure can be represented by $R[z, \Lambda_2]$. By virtue of this motion, the manipulator moves from the "home" configuration to the "intermediate" configuration.

3. Class X motion with $\phi_x = \Lambda_3$. The change in the orientation of the space structure can be represented by $R[x, \Lambda_3]$. At the end of this motion the manipulator returns to the "intermediate" configuration.
4. Class Z motion with $\phi_z = -\Lambda_2$, where Λ_2 is defined by Equation (3-8). The change in the orientation of the structure can be represented by $R[z, -\Lambda_2]$. By virtue of this motion, the manipulator moves from the "intermediate" to the "home" configuration.
5. Class Y motion with $\phi_y = \Lambda_4$. The change in the orientation of the structure can be represented by $R[y, \Lambda_4]$. At the end of this motion the manipulator returns to the "home" configuration.

If the manipulator goes through the sequence of motions discussed above, the change in the orientation of the space structure would be represented by the rotation matrix,

$$R[y, \Lambda_4] R[z, -\Lambda_2] R[x, \Lambda_3] R[z, \Lambda_2] R[y, \Lambda_1] \quad (3-9)$$

If any arbitrary change in the orientation of the space structure given by Equation (3-7) is to be attained through the above sequence of motions, then we should be able to solve for Λ_1 , Λ_3 , and Λ_4 from the following equation;

$$R[y, \Lambda_4] R[z, -\Lambda_2] R[x, \Lambda_3] R[z, \Lambda_2] R[y, \Lambda_1] = \quad (3-10)$$

$$R[y, \phi_3] R[x, \phi_2] R[y, \phi_1]$$

for arbitrary values of ϕ_1 , ϕ_2 , and ϕ_3 . Equation (3-10) will have a singularity for $\phi_2 = 0, \pm\pi$. Then Equation (3-10) can be solved by setting $\Lambda_2 = \Lambda_3 = \Lambda_4 = 0$, and equating $\Lambda_1 = (\phi_1 + \phi_3)$ when $\phi_2 = 0$, and $\Lambda_1 = (\phi_1 - \phi_3)$ when $\phi_2 = \pm\pi$. When $\phi_2 \neq 0, \pm\pi$, we solve for Λ_1 , Λ_3 , and Λ_4 by first rewriting Equation (3-10) as:

$$R[z, -\Lambda_2] R[x, \Lambda_3] R[z, \Lambda_2] = R[y, \phi_3 - \Lambda_4] R[x, \phi_2] R[y, \phi_1 - \Lambda_1] \quad (3-11)$$

$$\phi_2 \neq 0, \pm\pi$$

The product of the matrices on both sides of Equation (3-11) is in a direction cosine matrix that can be equivalently represented by the set of four euler parameters β_1 , β_2 , β_3 , and β_4 [Ref. 10] as follows:

$$\beta_0 = \cos\left(\frac{\Lambda_3}{2}\right), \quad \beta_0 = \cos\left(\frac{\phi_2}{2}\right) \cos\left(\frac{\phi_1 - \Lambda_1 + \phi_3 - \Lambda_4}{2}\right) \quad (3-12)$$

$$\beta_1 = \sin\left(\frac{\Lambda_3}{2}\right) \cos\Lambda_2, \quad \beta_1 = \sin\left(\frac{\phi_2}{2}\right) \cos\left(\frac{\phi_3 - \Lambda_4 - \phi_1 + \Lambda_1}{2}\right) \quad (3-13)$$

$$\beta_2 = \sin\left(\frac{\Lambda_3}{2}\right) \sin\Lambda_2, \quad \beta_2 = \cos\left(\frac{\phi_2}{2}\right) \sin\left(\frac{\phi_1 - \Lambda_1 + \phi_3 - \Lambda_4}{2}\right) \quad (3-14)$$

$$\beta_3 = 0, \quad \beta_3 = \sin\left(\frac{\phi_2}{2}\right) \sin\left(\frac{\phi_3 - \Lambda_4 - \phi_1 + \Lambda_1}{2}\right) \quad (3-15)$$

Since $\phi_2 \neq 0, \pm\pi$, Equations (3-12) through (3-15) can be solved for Λ_1 , Λ_3 , and Λ_4 as follows:

$$\Lambda_1 = \phi_1 - \arctan[\sin\Lambda_2 \tan(\Lambda_3/2)] , \quad (3-16)$$

$$\Lambda_3 = 2\arcsin[\sin(\phi_2/2)\sec\Lambda_2] , \quad (3-17)$$

$$\Lambda_4 = \Lambda_1 + \phi_3 - \phi_1 . \quad (3-18)$$

The algorithm for the reorientation of the space structure can, therefore, be established as follows: First solve for the necessary change in the orientation ϕ_1 , ϕ_2 , and ϕ_3 from Equation (3-7). Next compute the values of Λ_1 , Λ_3 , and Λ_4 from Equations (3-16) through (3-18) using the computed values of ϕ_1 , ϕ_2 and ϕ_3 . For each of Λ_1 , Λ_3 and Λ_4 , compute the closed trajectory in the θ_2 - θ_3 plane and the number of times that the manipulator has to traverse the closed trajectory. Such trajectories can always be planned. Now that the trajectories are known we can follow the five step motion sequence to achieve the desired change in orientation of the structure.

D. SIMULATION RESULTS

A simulation was conducted for a large angle maneuver of an arbitrary space structure where the manipulator had the following kinematic parameters, according to Figure 3.1b

$$r = 0.15 \text{ m}, l_1 = 0.35 \text{ m}, l_2 = 0.50 \text{ m}, l_3 = 0.40 \text{ m}$$

The dynamic parameters used are given in Table 3.1. The initial and desired orientations of the space structure, given in degrees were:

$$(\phi_{1i}, \phi_{2i}, \phi_{3i}) \equiv (135.0, 25.0, -105.0) \quad (3-19)$$

$$(\phi_{1f}, \phi_{2f}, \phi_{3f}) \equiv (-55.0, 95.0, 75.0) \quad (3-20)$$

This yielded the following y-x-y Euler angles:

$$\begin{aligned} \Lambda_1 &= -66.79483, \\ \Lambda_2 &= -11.09346, \\ \Lambda_3 &= 123.43739, \\ \Lambda_4 &= 89.83769 \end{aligned} \quad (3-21)$$

From the orientation and Euler angles, Λ_1 , Λ_2 , Λ_3 and Λ_4 were obtained as follows:

$$(\phi_1, \phi_2, \phi_3) \equiv (-86.47308, 119.57773, 70.15945) \quad (3-22)$$

where the units are in degrees.

The orientation of the structure at the beginning and the end of each of the five sequences of rotations is as shown in Figure 3.5. The description of the closed loop path in the θ_1 - θ_2 - θ_3 space is as shown in Figure 3.6. As the manipulator traces out the path, as described in Figure 3.6, the evolution

of the Euler angles with respect to time is illustrated in Figure 3.7.

1. Class Y motion with $\Lambda_1 = -66.79483$ degrees. The minimum number of times the robot has to move along a closed trajectory will be $n = 9$. Then, for each closed loop motion the change in orientation needs to be $-66.79483/9 = -7.42164$ degrees from Figure 3.4, we find that $\phi_y = +7.42164$ degrees corresponds to a value of θ_{2u} that lies between 125.0 and 135.0 degrees. Using these values as the lower and upper limits, we find the exact solution for $\phi_y = 7.42164$ in Equation (3-3) to be 133.84235 degrees. The negative sign in the change in orientation can be taken care of by simply travelling along the closed path in the negative direction.

In Figure 3.6 ABCDA denotes the directed closed path in the θ_2 - θ_3 plane. The change in the y-x-y Euler angles (ϕ_1, ϕ_2, ϕ_3) is shown in Figure 3.7 during the time $t = 0$ seconds to $t = 483.92$ seconds. It can be seen from Figure 3.7 that during this time ϕ_1 and ϕ_2 remain constant, whereas ϕ_3 changes with a periodic motion. The number of periods is equal to nine and corresponds to the number of times the second and third joints of the robot move along the closed path ABCDA in Figure 3.6. The configuration of the system at the start and finish of this motion is shown in Figure 3.5(a) and(b).

2. Class Z motion with $\Lambda_2 = -11.09346$ degrees. In Figure 3.6, the path segment AO corresponds to the motion. The variation of the y-x-y Euler angles during this motion are not very clear from Figure 3.7 because this motion takes only 10 seconds to complete, as compared to the total time of simulation which is of the order of 2166 seconds. Figure 3.5(b) and (c) show us the configuration of the system at the beginning and the end of this motion.
3. Class X motion with $\Lambda_3 = 123.43739$ degrees. The minimum number of times the robot has to move along a closed trajectory will be $n = 19$. Therefore, for each closed loop motion the change in the orientation needs to be $123.43739/19 = 6.49670$ degrees. From Figure 3.4, we find that $\phi_x = -6.49670$ degrees corresponds to a value of θ_{2u} that lies between 125.0 and 135.0 degrees. Using these values as the lower and upper limits we find the exact solution for $\phi_x = -6.49670$ to be 132.19918 degrees. Since travelling along the positive direction of the closed path produces a negative change in the orientation

ϕ_x as evident from Figure 3.4, we will travel in the negative direction. In Figure 3.6, *OPQRO* denotes the directed closed loop path in the θ_2 - θ_3 plane. The change in the y-x-y Euler angles (ϕ_1, ϕ_2, ϕ_3) is shown in Figure 3.7 during the time $t = 493.92$ seconds to $t = 1509.27$ seconds. It can be seen from the figure that all the Euler angles undergo a periodic motion during this time. The number of periods can be seen to be equal to nineteen and equals the number of times the second and third joints of the robot move along the closed path *OPQRO* in Figure 3.6. The configuration of the system at the start and finish of this motion is shown in Figure 3.5(c) and (d).

4. Class Z motion with $\Lambda_2 = 11.09346$ degrees. In Figure 3.6, the path segment *OA* corresponds to this motion. The variation of the y-x-y Euler angles during this motion are not very clear from Figure 3.7 because this motion takes only 10 seconds to complete, as compared to the total simulation time which is of the order of 2166 seconds. Figure 3.5(d) and (e) show us the configuration of the system at the beginning and end of this motion.
5. Class Y motion with $\Lambda_1 = 89.83769$ degrees. The minimum number of times the robot has to move along a closed trajectory will be $n = 12$. Then, for each closed loop motion the change in the orientation needs to be $89.83769/12 = 7.48647$ degrees. From Figure 3.4, we find that $\phi_y = 7.48647$ degrees corresponds to a value of θ_{2u} that lies between 125.0 and 135.0 degrees. Using these values as lower and upper limits, we find the exact solution for $\phi_y = 7.48647$ in Equation (3-3) to be 134.73799 degrees. In Figure 3.6, *AMNBA* denotes the directed closed loop path in the θ_2 - θ_3 plane. The change in the y-x-y Euler angles (ϕ_1, ϕ_2, ϕ_3) is shown in Figure 3.7 during the time $t = 1519.27$ seconds to $t = 2166.64$ seconds. It can be seen from the figure that during this time the Euler angles ϕ_1 and ϕ_2 remain constant whereas ϕ_3 changes with a periodic motion. The number of periods can be seen to be equal to twelve and it equals the number of times the second and third joints of the robot move along the closed path *AMNBA* in Figure 3.6. The configuration of the system at the start and finish of this motion is shown in Figure 3.5(e) and (f).

We have thus effectively demonstrated a new method for attitude control of a freely-floating space structure via a surface integral approach. The next chapter will address the

issue of how to maneuver a manipulator without effecting an overall change in the orientation of the space structure.

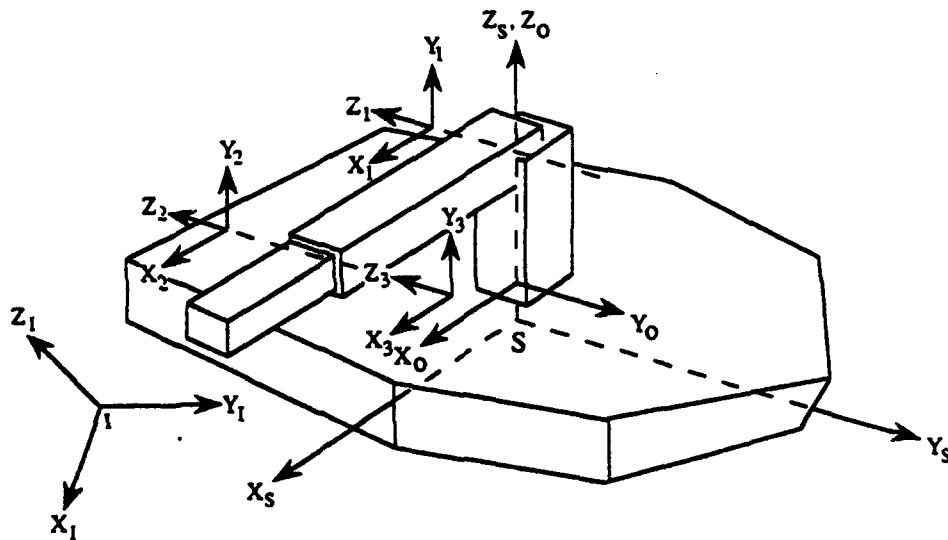


Figure 3.1(a): The Home Configuration of the Three Link Robot Manipulator Mounted on the Space Structure is Shown. The Link Frames are According to the Denavit-Hartenberg Convention.

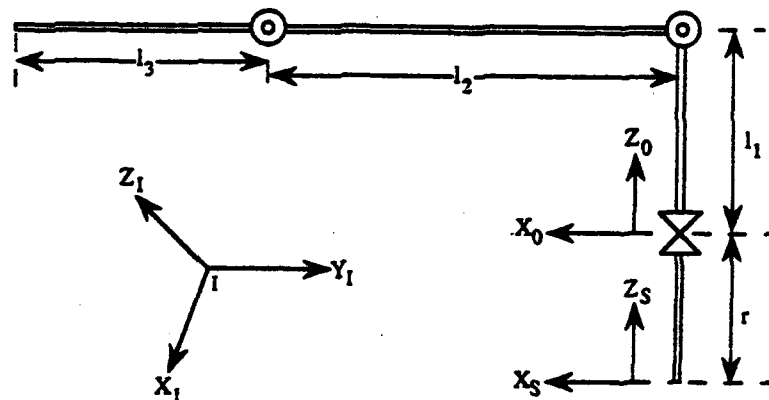
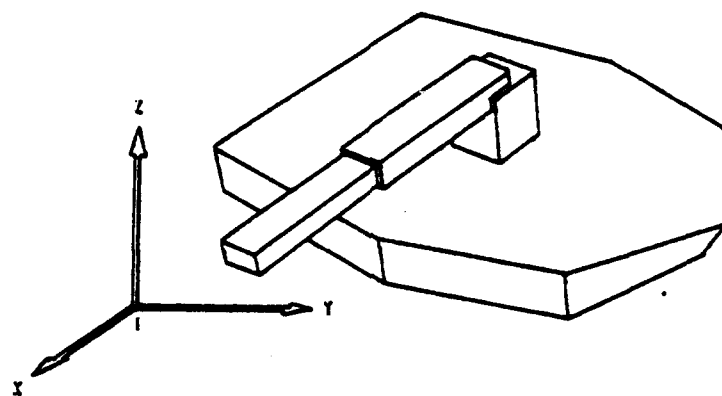
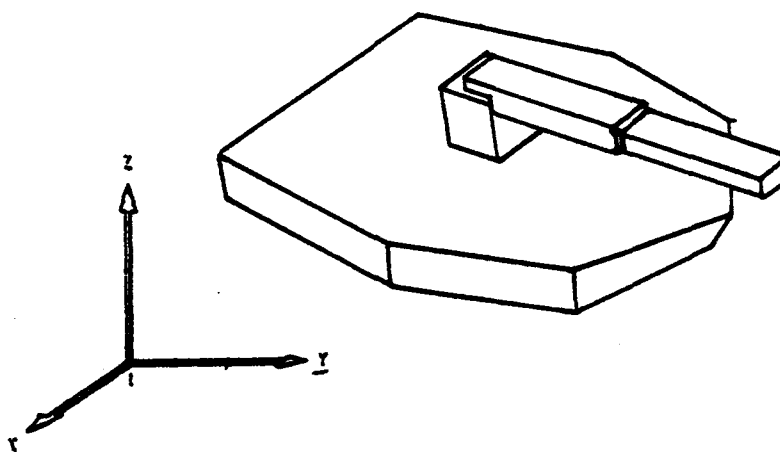


Figure 3.1(b): Kinematic Structure of the 3-Link Robot Manipulator with Revolute Joints. The Home Configuration of the Manipulator is Shown.

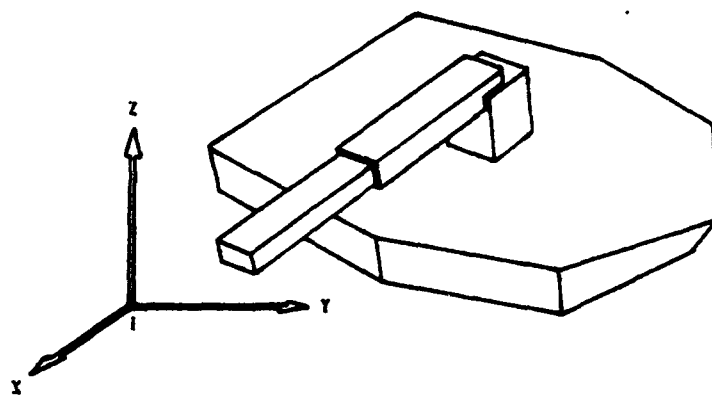


(a)

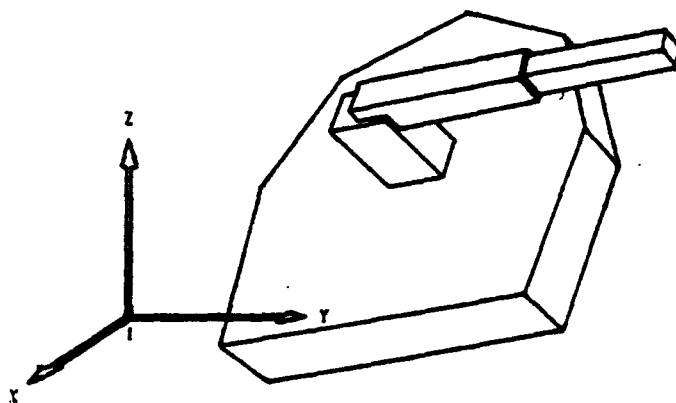


(b)

Figure 3.2: Satellite Configurations (a) Home Configuration with $\theta_1, -\theta_2 = \theta_3 = 0$. (b) Intermediate Configuration with $\theta_1 = 90^\circ$, $\theta_2 = \theta_3 = 0^\circ$.



(a)



(b)

Figure 3.3: The (a) Initial and (b) Final Configurations of the Space Structure.

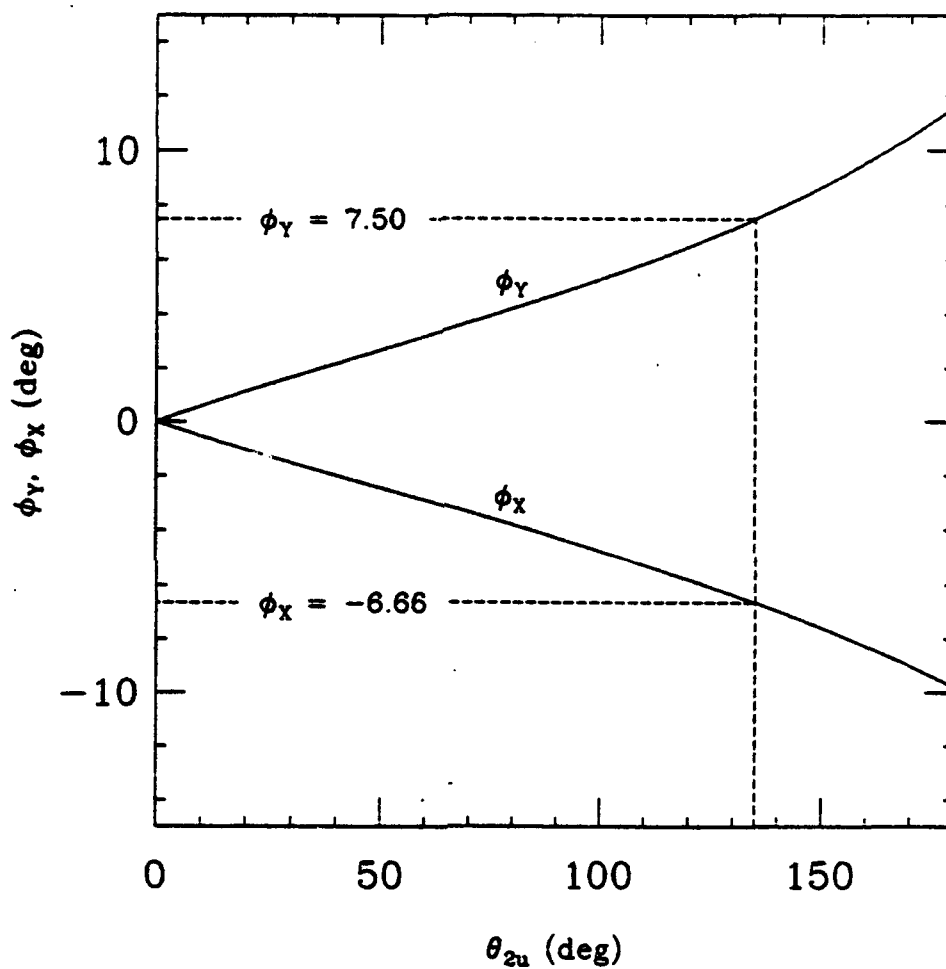


Figure 3.4: For the Simulation in Section D, the Change in the Orientation of the Space Structure about its x and y axes: ϕ_x and ϕ_y respectively, Depends Upon the Dimension of the Rectangular Path in the θ_2 - θ_3 Plane.

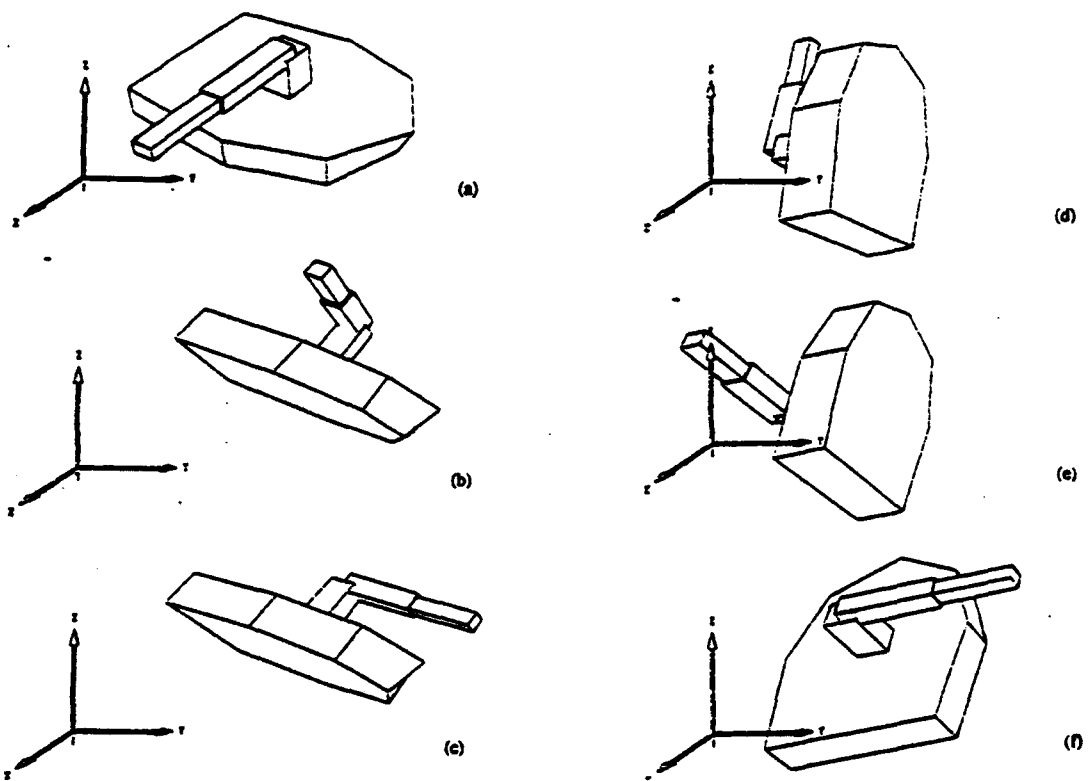


Figure 3.5: Initial, Intermediate and Final Configurations of the System.

TABLE 3.1: DYNAMIC PARAMETERS OF THE 3-R MANIPULATOR
 I_{kij} (kg-m²).

	k=0	k=1	k=2	k=3
(1,1)	23.95781	00.0830	00.0147	00.0117
(2,2)	13.87031	00.0103	00.2343	00.1221
(3,3)	37.82812	00.0830	00.2343	00.1221
m_i (kg)	302.6250	7.62615	10.8945	8.71560

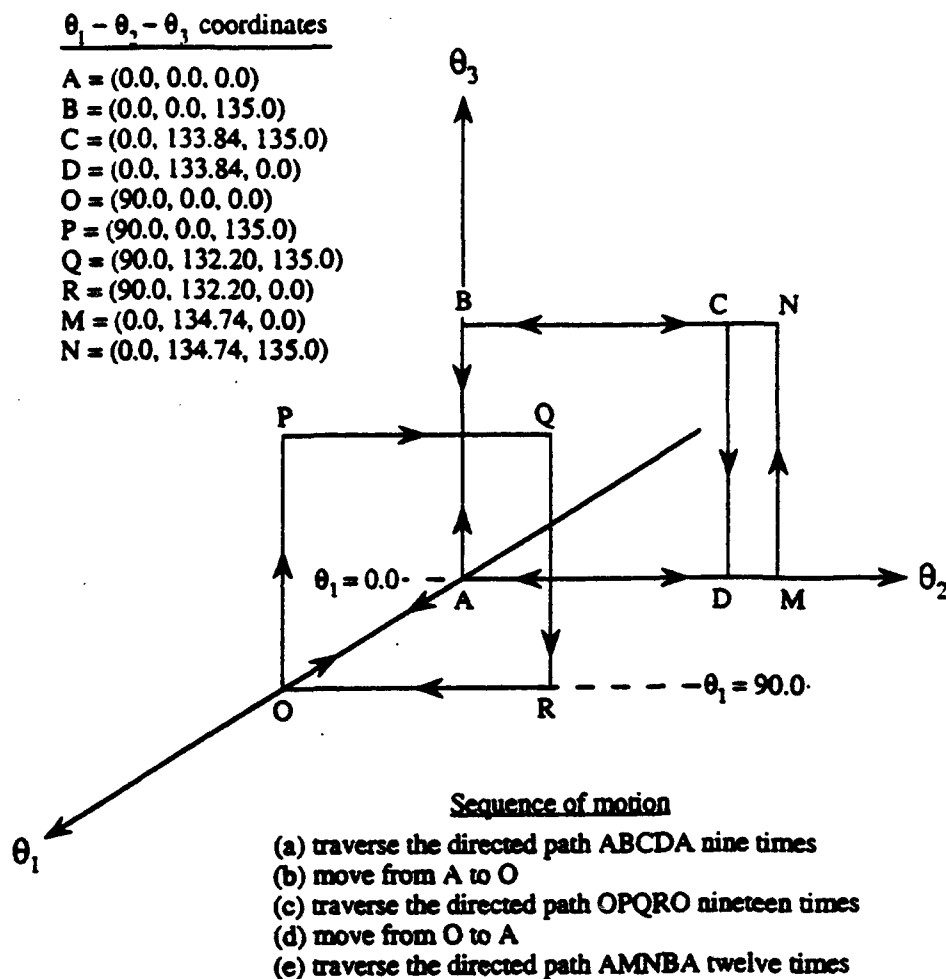


Figure 3.6: Description of the Closed Loop Path in $\theta_1 - \theta_2 - \theta_3$ Space that Changes the Orientation of the Space Structure from an Initial y-x-y Euler Angles of (135.0, 25.0, -105.0) Degrees to a Final Value of (-55.0-95.0, 75.0) Degrees.

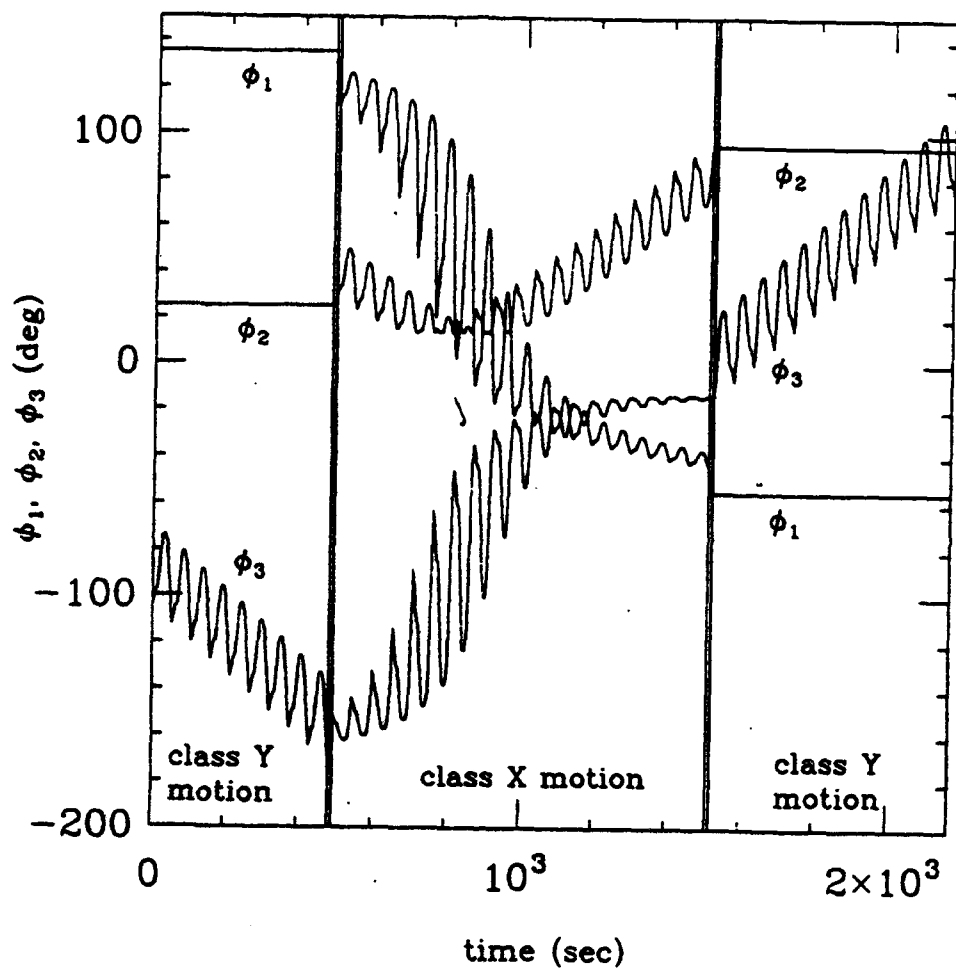


Figure 3.7: Evolution of the Euler Angles ϕ_1 , ϕ_2 , ϕ_3 Describing the Orientation of the Space Structure, for the Simulation in Section D.

IV. -- PLANNING REPEATABLE PATHS FOR PLANAR SPACE ROBOTS

A. INTRODUCTION

In Chapters I and III, we saw that due to the nonholonomic nature of a freely-floating space structure the use of an organic manipulator will result in a change in the orientation of the structure. Automation in space requires the ability for a space robot to perform a task repeatedly in its work space without any drift in its configuration variables, i.e., joint angles, orientation, and end-effector position. Hence, the resultant change in orientation of the structure as the manipulator arm performs a required task, which we exploited in Chapter III for attitude control, is undesirable for automation.

In Chapter II we proposed: (1) that integrability of the differential constraint is only a sufficient condition for repeatability, but it is by no means a necessary condition, and (2) that a necessary condition for repeatable motion was that the function F defined by Equation (2-10) be equal to zero.

This chapter will apply these ideas to a two dimensional space robot. The two dimensional case is studied purely for its simplicity.

B. NECESSARY CONDITION FOR REPEATABILITY

Not all nonholonomic systems exhibit pseudo-holonomic behavior. Consider the rolling disk shown in Figure 4.1 [Ref. 8]. The two nonholonomic constraints are given by:

$$dx - r \sin \alpha d\theta \quad (4-1a)$$

$$dy - r \cos \alpha d\theta \quad (4-1b)$$

Rearranging Equation(4-1) for the change in the dependent variables x and y for the closed loop motion of the independent variables θ and α we get:

$$\int dx = \int r \sin \alpha d\theta \quad (4-2a)$$

$$\int dy = \int r \cos \alpha d\theta \quad (4-2b)$$

where $F(\alpha, \theta) \triangleq \{ \sin \alpha, \cos \alpha \}$. Since $F(\alpha, \theta)$ will not equal zero at any point in the space of θ and α it will not satisfy the condition for repeatability, consequently it does not admit repeatable motion.

In the case of a planar space robot with two links, shown in Figure 4.2, the nonholonomic conservation of momentum constraint equation is given by Equation(4-3).

$$\begin{aligned} \omega = d\theta_0 &= -\left(\frac{B}{A}\right) d\theta_1 - \left(\frac{C}{A}\right) d\theta_2 \\ &= g_1(\theta_1, \theta_2) d\theta_1 + g_2(\theta_1, \theta_2) d\theta_2 \end{aligned} \quad (4-3)$$

where

θ_0 = the orientation of the space vehicle,

θ_1 and θ_2 = the joint variables of the manipulator,

A, B, C = functions of θ_1 and θ_2 as defined in Appendix A,

r = the position of the joint of the first link with respect to the center of gravity of the body,

l_1 = the length of the first link,

l = the length of the second link,

m_0, m_1, m_2 are the masses of the rigid body and the two links,

I_0, I_1, I_2 are the moments of inertia of the rigid body and the two links about their respective centers of mass,

$M = m_0 + m_1 + m_2$, and

$I_t = I_0 + I_1 + I_2$.

Applying Stoke's theorem to Equation (4-3) we get

$$\begin{aligned} \int d\theta_0 &= \iint_S \left[\frac{\partial}{\partial \theta_2} \left(\frac{B}{A} \right) - \frac{\partial}{\partial \theta_1} \left(\frac{C}{A} \right) \right] d\theta_1 d\theta_2 \\ &= \iint_S \left[\frac{A \frac{\partial B}{\partial \theta_2} - B \frac{\partial A}{\partial \theta_2} - A \frac{\partial C}{\partial \theta_1} + C \frac{\partial A}{\partial \theta_1}}{A^2} \right] d\theta_1 d\theta_2 \end{aligned} \quad (4-4)$$

where S is the region enclosed by the path C in the joint space of the robot which the manipulator arm traces. We can show that $A \neq 0$ therefore Equation (4-4) will satisfy the necessary condition for repeatability if;

$$F(\theta_1, \theta_2) \triangleq \left[A \frac{\partial B}{\partial \theta_2} - B \frac{\partial A}{\partial \theta_2} - A \frac{\partial C}{\partial \theta_1} + C \frac{\partial A}{\partial \theta_1} \right] = 0 \quad (4-5)$$

where $A^2 \neq 0$.

To find the "holonomic loop", on which the planar space robot exhibits holonomic behavior globally, we need find the values of θ_1 and θ_2 which set Equation (4-5) to zero. To determine the appropriate values of θ_1 and θ_2 , we choose path in the robot joint space, which we desire the robot to execute, such that it encloses at least one point where the function F goes to zero. This path can then be optimized by using a variety of numerical optimization techniques to drive Equation (4-5) to zero. In simulation we choose to use (1) an elliptical path as the most general case of a path; and, (2) the steepest descent optimization technique for its simplicity.

The elliptical path, shown in Figure 4.3, was parameterized as follows:

$$\theta_1 = \theta_{10} + a \cos\phi \cos 2\pi t - b \sin\phi \sin 2\pi t, \quad t \in [0,1] \quad (4-6a)$$

$$\theta_2 = \theta_{20} + a \sin\phi \cos 2\pi t + b \cos\phi \sin 2\pi t, \quad t \in [0,1] \quad (4-6b)$$

where a and b are the semi-major and semi-minor axes of the ellipse respectively, ϕ is the angle of inclination of the ellipse with the θ_1 axis, θ_{10} and θ_{20} are the coordinates of the center of the ellipse. Substituting Equation (4-6) and its time derivatives into Equation (4-4), $d\theta$ can be expressed as a function of the single variable t such that we get:

$$\begin{aligned}\int d\theta_0 &= \int (g_1 d\theta_1 + g_2 d\theta_2) \\ &= \int [g_1 \dot{\theta}_1 + g_2 \dot{\theta}_2] dt\end{aligned}\tag{4-7}$$

To optimize the path we need to: (1) arbitrarily choose the parameters, θ_{10} , θ_{20} , a , b , and ϕ , of the ellipse; and, (2) to change the five parameters so that the value of the surface integral given by Equation (4-7) is equal to zero.

In making the initial choices of the ellipse parameters, we needed to ensure that: (1) the ellipse encompasses at least one point where the function F defined by Equation (2-10) is equal to zero. This can be satisfied by considering Figure 4.4, which provides the set of all points where the function F vanishes; and, (2) the elliptical path lies in the work space of the robot. This can be done by applying the methods discussed in [Ref. 23].

For the optimization, to eliminate the trivial solution, where the surface integral is zero, because the area of the closed path is equal to zero, we imposed the restriction that the area of the ellipse was constant. In other words, a and b were not allowed to change independently of each other. This imposed the added constraint,

$$a db + b da = 0\tag{4-8}$$

We define a function V as follows:

$$V = \zeta^2, \quad \zeta = \int_D F(\theta_1, \theta_2) d\theta_1 d\theta_2 \quad (4-9)$$

and solve the unconstrained minimization problem by implicitly assuming that a and b are dependent.

The steepest descent method involved numerical partial differentiation to change the parameters of the ellipse and to solve the unconstrained minimization problem where;

$$d\theta_{10} = -\zeta \frac{\partial \zeta}{\partial \theta_{10}}, \quad (4-10a)$$

$$d\theta_{20} = -\zeta \frac{\partial \zeta}{\partial \theta_{20}}, \quad (4-10b)$$

$$d\phi = -\zeta \frac{\partial \zeta}{\partial \phi}, \quad (4-10c)$$

$$da = -\zeta \frac{\partial \zeta}{\partial a}. \quad (4-10d)$$

This provided a systematic way to reach the local minimum value of the function V. If this minimum value is zero, then we have found the "holonomic loop". Though in general, the method of steepest descent does not guarantee the convergence of a function to its global minimum value, in our case the method always converged to a minimum. This was due to the particular nature of the function F.

C. SIMULATION RESULTS

A simulation was conducted for a planar space robot which had the kinematic and dynamic parameters given in Table 4.1. The initial parameters of the ellipse were chosen to be:

$$a=1.50000, \quad b=1.00000,$$

$$\phi= 0.75000,$$

$$\theta_{10}=0.50000, \quad \theta_{20}=0.50000$$

The initial and optimal path parameters yielded the paths as shown in Figure 4.5. Path I and II correspond to the initial and optimized path parameters, respectively. Path I yielded the numerical value for Equation (4-9) of $\zeta = -0.162775$. The optimized path parameters were;

$$a=1.31117, \quad b=1.14381,$$

$$\phi=0.79302,$$

$$\theta_{10}=0.34094, \quad \theta_{20}=-0.07054$$

yielding $\zeta = -9.9636 \times 10^{-9}$. Note that the sinusoidal curve, $F(\theta_1, \theta_2) = 0$, inset in Figure 4.5 passes through both Paths I and II, therefore, both paths satisfy the necessary condition for repeatability. Several simulations were carried out and in all cases the "holonomic loops" were found.

By finding the "holonomic loop", control of the attitude and, hence, the end-effector of the manipulator was obtained. The drift in the end-effector of the manipulator for the original path and optimized path are given in Figures 4.6 and 4.7, respectively. The magnitude of the drift in the case of

Path I is 76.96 mm/cycle. The magnitude of the drift in the case of Path II is negligible at 0.87 mm/cycle.

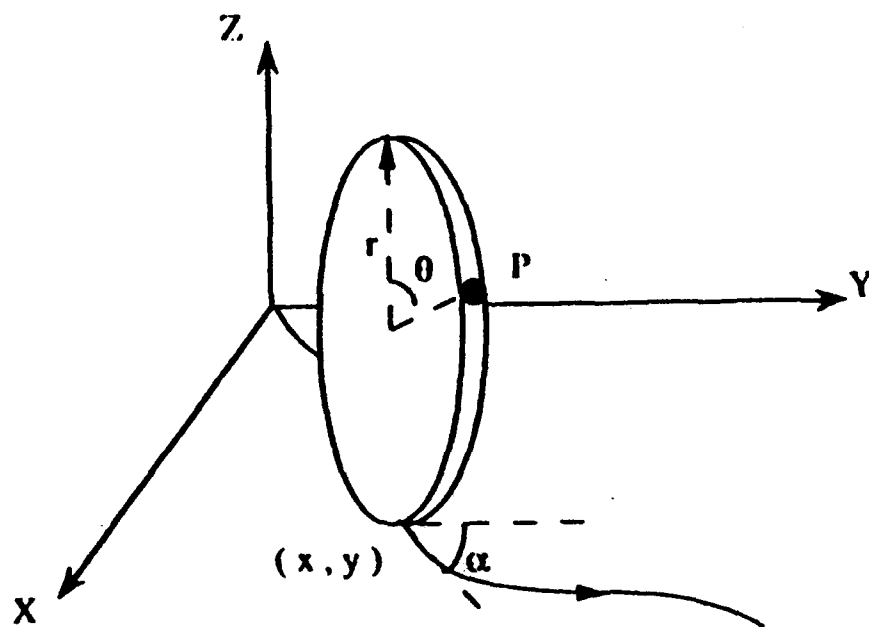


Figure 4.1: A Rolling Disk on a Flat Surface.

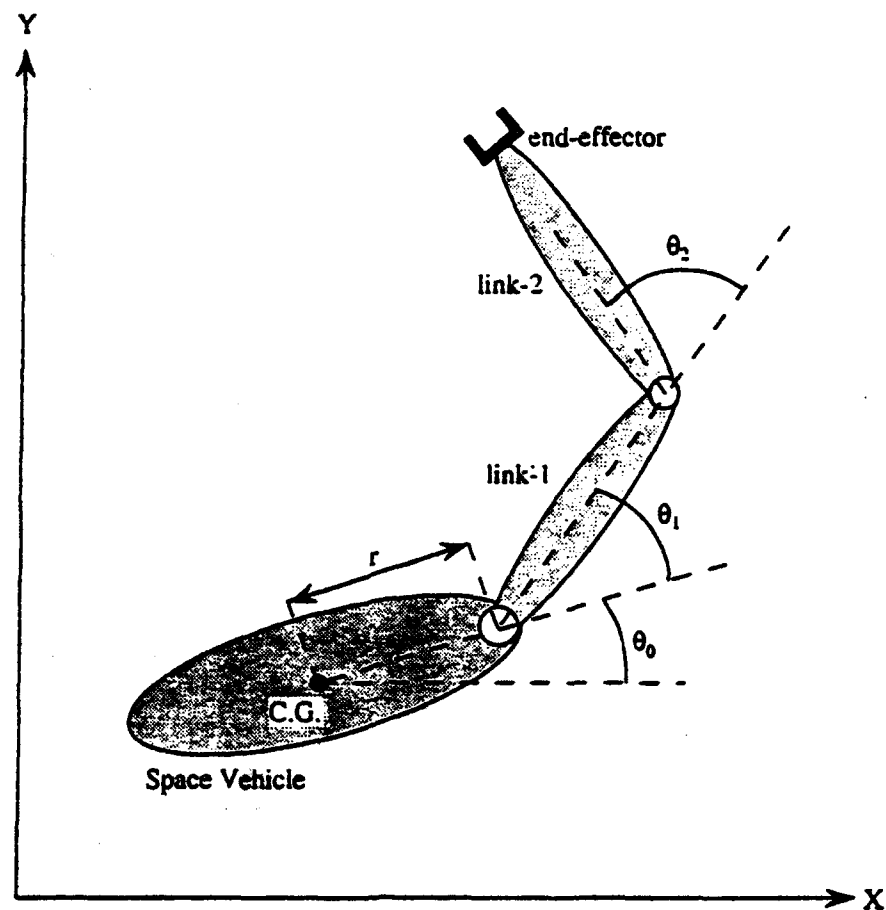


Figure 4.2

A Planar Space Robot with Two Links is Capable of Exhibiting Pseudo-holonomic Behavior.

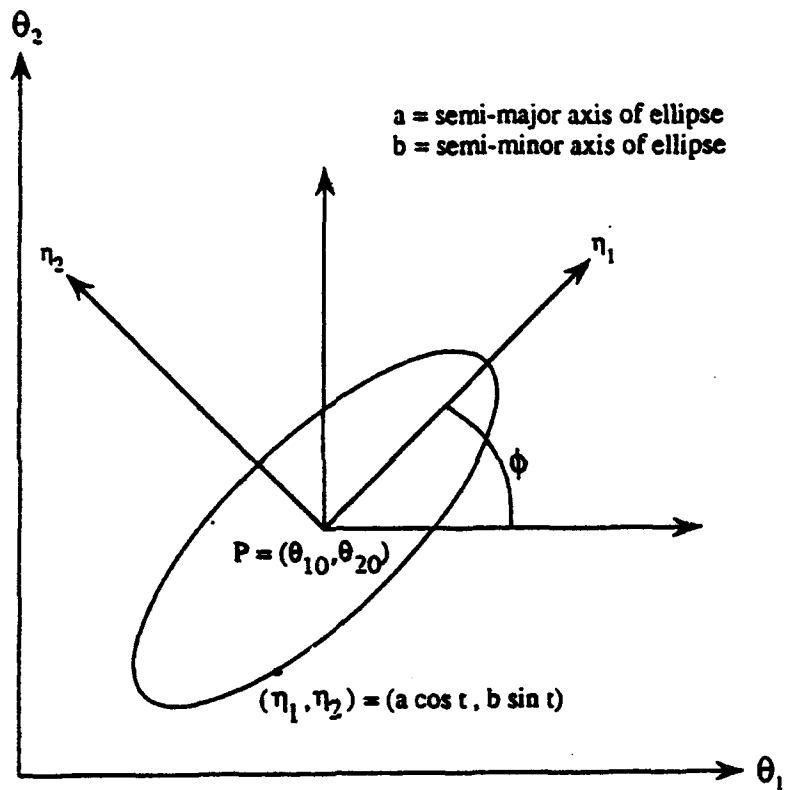


Figure 4.3

Parametric Representation of the Elliptical Path in the Joint Space of the Robot. P is the Center of the Ellipse, and ϕ is the Angle Between the Major Axis of the Ellipse and θ_1 .

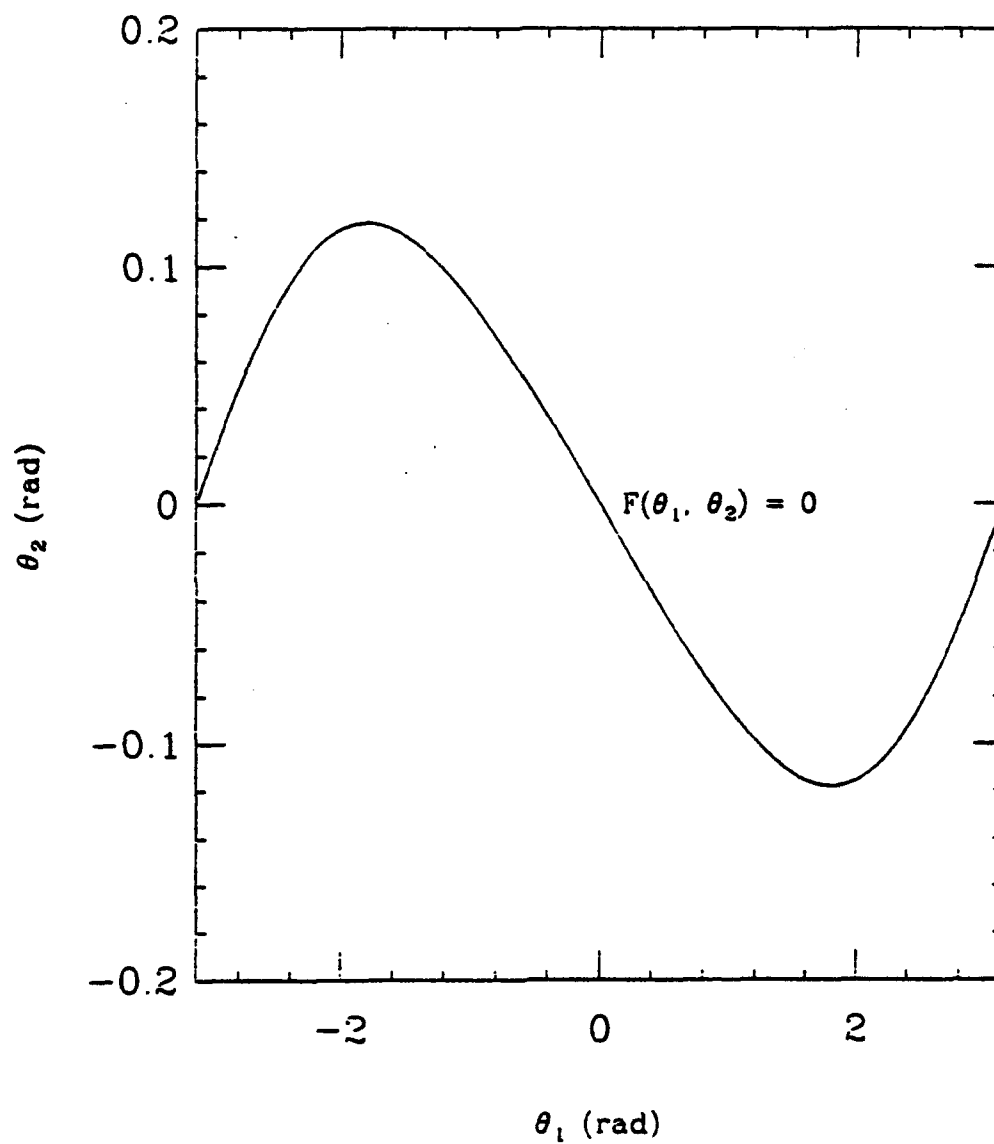


Figure 4.4 All Points in θ_1 - θ_2 Where $F(\theta_1, \theta_2) = 0$.

TABLE 4.1: KINEMATIC AND DYNAMIC PARAMETERS

	Mass	Inertia	Length
	(kg)	(kg-m²)	(m)
Vehicle	27.440	1.520	$r = 0.20$
Link-1	5.380	0.115	$l_1 = 0.50$
Link-2	2.640	0.028	$l_2 = 0.35$

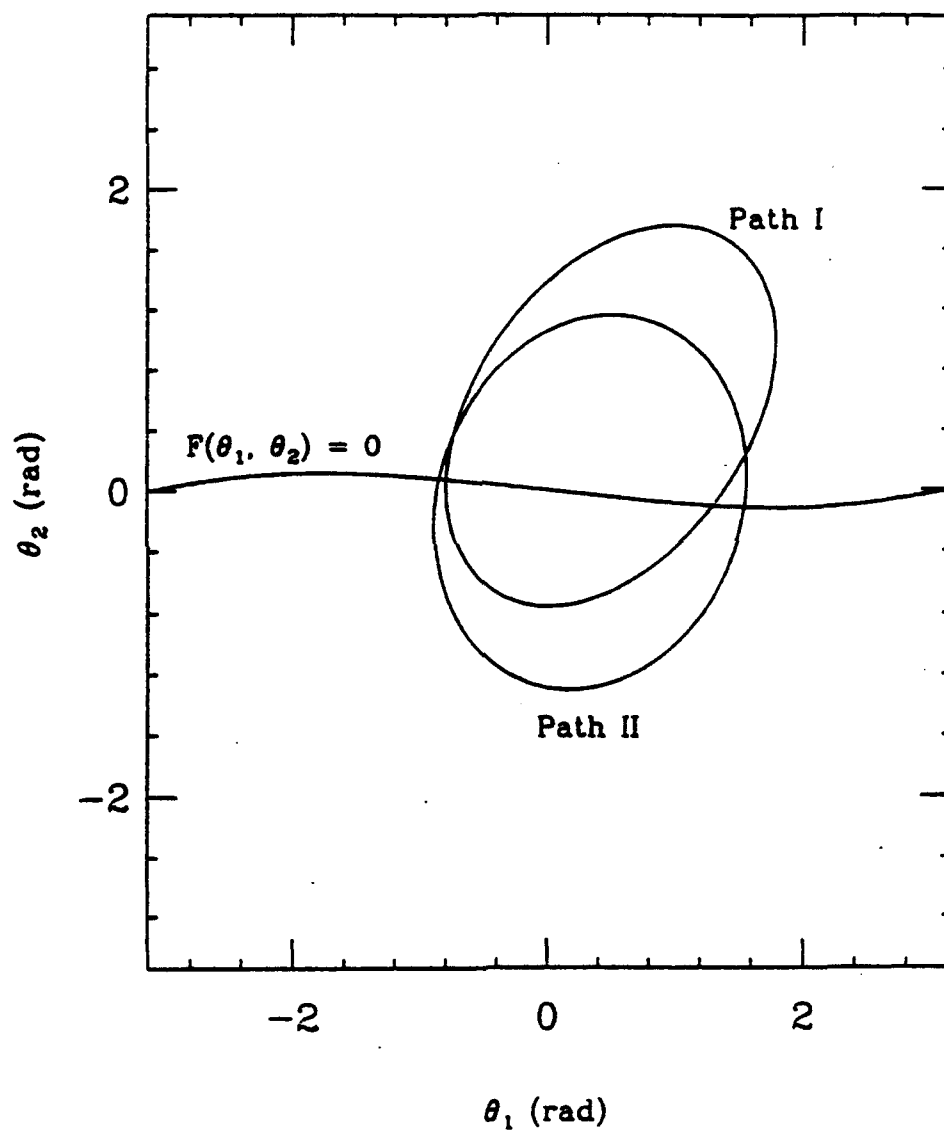


Figure 4.5 **Elliptical Paths in Joint Space.**

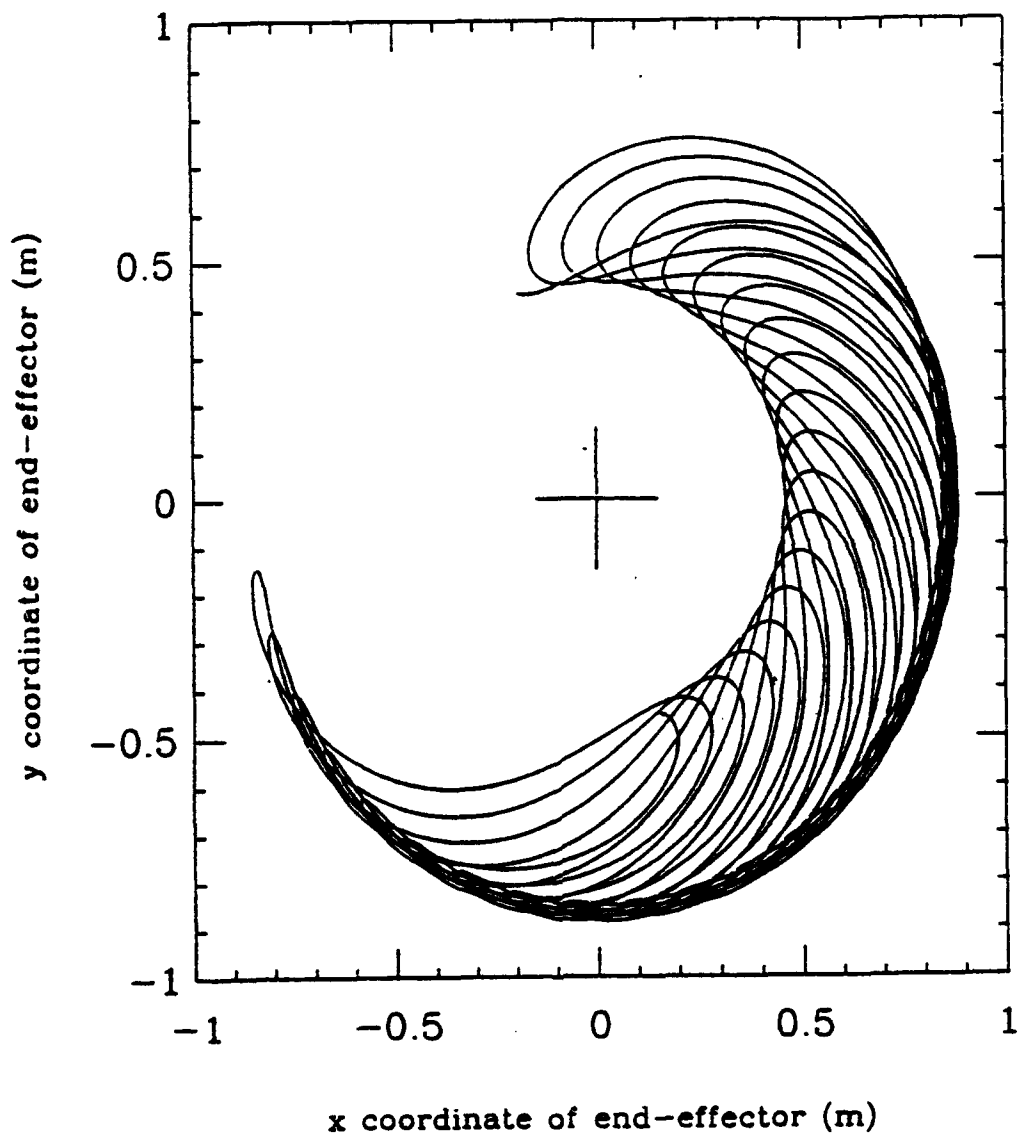


Figure 4.6 End-Effector Drift in 20 Cycles for Path I.

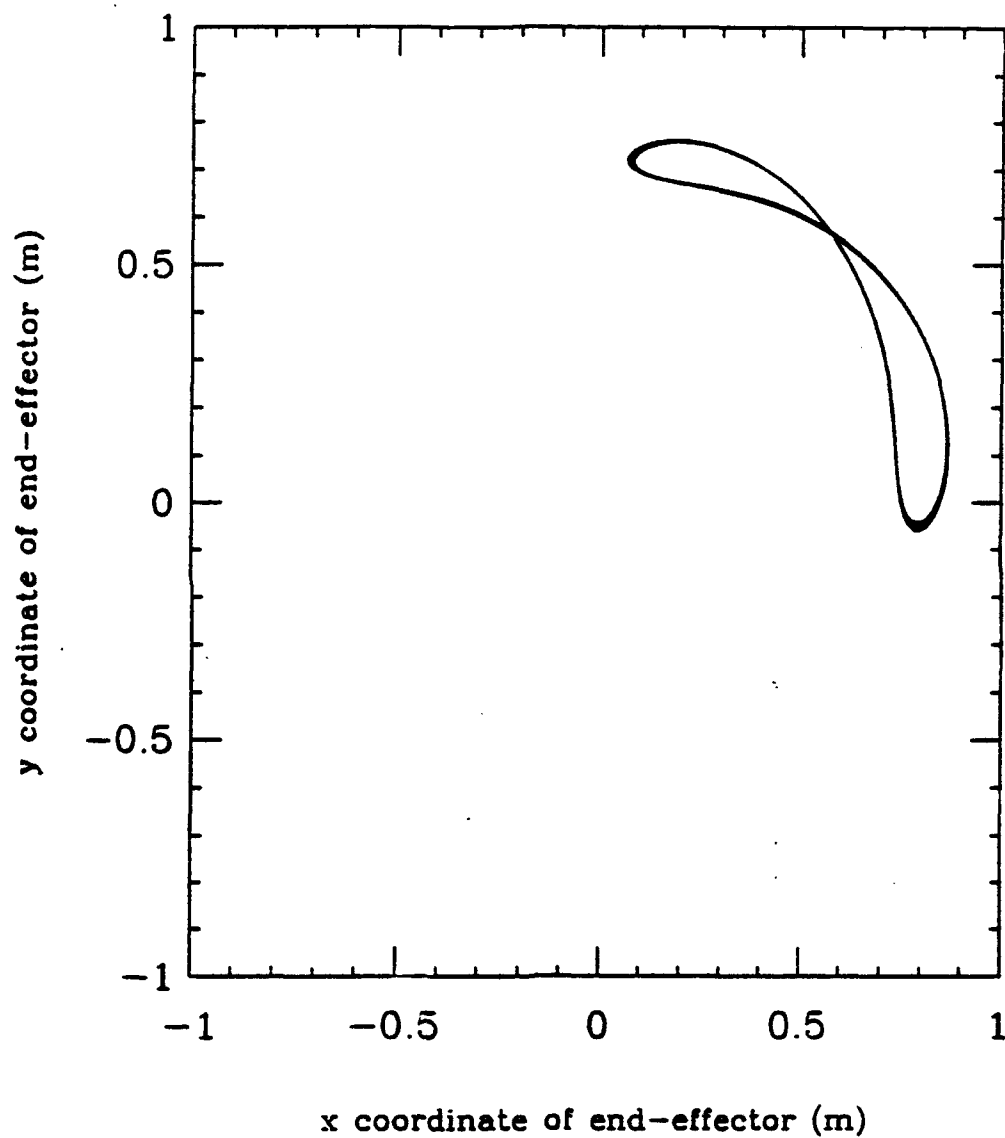


Figure 4.7 Repeatable End-Effector Motion for Path II.

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

This thesis has presented two concepts. The first being a algorithm for the motion planning of a space manipulator to achieve attitude control of a freely-floating three dimensional space structure. Generally stated the algorithm provided a means for calculating the coordinate trajectories required to drive a nonholonomic system from one point in its configuration space to some other desired point. The algorithm invoked Stokes' Theorem and hence took a surface integral approach to the motion planning problem. In particular, we considered a three dimensional structure with a three link manipulator arm.

Due to the nonholonomic nature of structures in space, articulated joint manipulators can effectively be used as a back-up means to a gyroscope for the attitude control of these structures. Attitude control is achieved through the motion planning of the internal motion of the manipulator arm joints. We found the surface integral approach to be a simple and effective means to solving the motion planning problem.

Secondly, we presented a means of determining the manipulator motion required for the nonholonomic freely-floating space structure to behave in a holonomic manner,

which we called "pseudo-holonomic". Our method determined if "holonomic loops" existed, where the system exhibits holonomic behavior globally for the configuration space of the nonholonomic system. If a "holonomic loop" did exist we presented an algorithm for finding that loop within the configuration space. In this case, we looked at a planar space robot with a two link manipulator arm.

Additionally, though the nonholonomic nature of the space structure does not normally admit repeatable motion, it is possible, however, to find exceptions to the rule where systems do exhibit holonomic behavior globally. Finding the "holonomic loop" in the joint space of the manipulator admitted repeatable motion of the space robot. Hence, we have seen that manipulators can effectively serve the dual purpose of attitude control and automation in space.

We have demonstrated the ability to predict and control the change in orientation of a freely-floating space structure.

B. RECOMMENDATIONS

The application of the two algorithms to more complicated structures is the next logical step. The approach presented here can be extended to other nonholonomic systems such as mobile robots. The finding of "holonomic loops" can be further extended to the three dimensional space structure with an attached three linked manipulator arm.

APPENDIX A.

The terms A , B , and C in Eq.(2) are defined as follows

$$A \triangleq I_0 + \frac{1}{M} r^2 m_0 (m_1 + m_2) + \frac{l_1^2}{4M} (m_0 m_1 + m_1 m_2 + 4m_0 m_2) + \frac{l_2^2}{4M} m_2 (m_0 + m_1) \\ + \frac{1}{M} m_0 (m_1 + 2m_2) r l_1 \cos \theta_1 + \frac{1}{M} m_2 (m_0 + 0.5m_1) l_1 l_2 \cos \theta_2 + \frac{1}{M} m_0 m_2 r l_2 \cos(\theta_1 + \theta_2)$$

$$B \triangleq I_1 + I_2 + \frac{l_1^2}{4M} (m_0 m_1 + m_1 m_2 + m_0 m_2) + \frac{l_2^2}{4M} m_2 (m_0 + m_1) + \frac{2}{M} m_0 (m_1 + 2m_2) r l_1 \cos \theta_1 \\ + \frac{1}{M} m_2 (m_0 + 0.5m_1) l_1 l_2 \cos \theta_2 + \frac{1}{2M} m_0 m_2 r l_2 \cos(\theta_1 + \theta_2)$$

$$C \triangleq I_2 + \frac{l_2^2}{4M} m_2 (m_0 + m_1) + \frac{1}{2M} m_0 m_2 l_1 l_2 \cos \theta_2 + \frac{1}{2M} m_0 m_2 r l_2 \cos(\theta_1 + \theta_2)$$

where, m_0 , m_1 , and m_2 are the masses of the space vehicle and the two links of the manipulator, I_0 , I_1 , and I_2 are the moment of inertias of the space vehicle and the two links about their center of masses, r is the distance of the first joint from the center of mass of the vehicle, l_1 and l_2 are the lengths of the two links, $M \triangleq m_0 + m_1 + m_2$, and $I_0 \triangleq I_0 + I_1 + I_2$.

LIST OF REFERENCES

1. Alexander, H. L., and Cannon, R. H., 1987, "Experiments on the Control of a Satellite Mounted Manipulator", Proc. American Control Conference.
2. Arnold, V. I., 1989, "Mathematical Methods of Classical Mechanics", Springer Verlag.
3. Denavit, J., and Hartenberg, R. S., 1955, "A Kinematic Notation for Lower Pair Mechanisms Based on Matrices", Journal of Applied Mechanics, p. 22.
4. Fernandes, C., Gurvits, L., and Li, Z. X., 1991, "Foundations of Nonholonomic Motion Planning", Technical Report No. 577, Robotics Research Laboratory, Courant Institute of Mathematical Sciences.
5. Finney, R., Thomas G., 1990, "Calculus", Addison-Wesley Publishing Company.
6. Fletcher, R., and Reeves, C. M., 1964, "Function Minimization by Conjugate Gradients", "Computer Journal", Vol. 7, No. 2, pp. 149-154.
7. Gradshteyn, I. S., and Ryzhik, I. M., 1981, "Table of Integrals, Series, and Products", Academic Press, New York.
8. Greenwood, D. T., 1988, "Principles of Dynamics", Prentice Hall, Englewood Cliffs, New Jersey.
9. Ince, E. L., 1956, "Ordinary Differential Equations", Dover Publications, New York.
10. Junkins, J. L., and Turner, T. D., 1986, "Optimal Spacecraft Rotational Maneuvers", Elsevier, New York.
11. Kane, T. R., Hendrick, M. R., and Yatteau, J. D., 1972, "Experimental Investigation of an Astronaut Maneuvering Scheme", Journal of Biomechanics, Vol. 5, pp. 313-320.
12. Kane, T. R., and Scher, M. P., 1969, "A Dynamical Explanation of the Falling Cat Problem", International Journal of Solids and Structures, Vol. 5, pp. 663-670.

13. Klein, C. A., Huang, C., "Review of Pseudoinverse Control for Use with Kinematically Redundant Manipulators", IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-13, No. 13, pp. 245-250.
14. Kreyzig, E., 1972, "Advanced Engineering Mathematics", Willey Eastern Limited.
15. Krishnan, H., McClamroch, N. H., Reyhanoglu, M., 1992, "On the Attitude Stabilization of a Rigid Spacecraft Using Two Central Torques", Proc. of the American Control Conference, pp. 1990-1995.
16. Longman, R. W., Lindberg, R. E., and Zedd, M. F., "Satellite Mounted Robot Manipulators: New Kinematics and Reaction Moment Compensation", International Journal of Robotics Research, Vol. 6, No. 3, pp. 87-103.
17. Luo, S., Ahmad, S., "Predicting the Drift Motion for Kinematically Redundant Robots", IEEE Transactions on Systems, Man, and Cybernetics, Vol. 22, No. 4, pp. 717-728.
18. Miyazaki, F., Masutani, Y., and Arimoto, S., 1988, "Sensor Feedback Using Approximate Jacobian", UAS-Japan Symposium on Flexible Automation, pp.139-145.
19. Mukherjee, R., and Anderson, D., 1992, "A Surface Integral Algorithm for the Motion Planning of Nonholonomic Mechanical Systems", Master's thesis, Naval Postgraduate School, Monterey, California.
20. Mukherjee, R., and Zurewsky, M., 1993, "Reorientation of a Structure in Space Using a Three Link Rigid Manipulator", AIAA Conference on Guidance, Control and Navigation, Monterey, California.
21. Mukherjee, R., and Zurewsky, M., 1993, "Pseudo-Holonomic Behavior of Planar Space Robots", technical brief submitted to the ASME Journal of Dynamic Systems, Measurement and Control.
22. Nakamura, Y., and Mukherjee, R., 1991, "Nonholonomic Motion Planning of Space Robots via a Bi-Directional Approach", IEEE Transactions on Robotics and Automation, Vol. 7, No. 4, pp. 500-514.
23. Papadopoulos, E., and Dubowsky, S., 1989, "On the Dynamic Singularities in the Control of Free-Floating Space Manipulators", ASME Winter Annual Meeting: Dynamics and

Control of Multibody/Robotic Systems with Space Applications, DSC-Vol. 15, pp. 45-51.

24. Reyhanoglu, M., and McClamroch, N. H., 1992, "Planar Reorientation Maneuvers of Space Multibody Systems Using Internal Controls", AIAA Journal of Guidance, Control, and Dynamics, Vol. 15, No. 6, pp. 1475-1480.
25. Reyhanoglu, M., McClamroch, N. H., 1991, "Controllability and Stabilizability of Planar Multibody Systems with Angular Momentum Preserving Control Torques", Proc. of the American Control Conference, Boston, Massachusetts.
26. Shamir, T., Yomdin, Y., "Repeatability of Redundant Manipulators: Mathematical Solution of the Problem" IEEE Transactions on Automatic Control, Vol. 33, No. 11, pp. 1004-1009.
27. Sreenath, N., 1992, "Nonlinear Control of Planar Multibody Systems in Shape Space", Mathematics of Control, Signals and Systems, Vol. 5, pp. 343-363.
28. Umetani, Y., and Yoshida, K., 1987, "Continuous Path Control of Space Manipulators Mounted on OMV", Acta Astronautica, Vol. 15, No. 12, pp. 981-986.
29. Vafa, Z., and Dubowsky, S., 1987, "On the Dynamics of Manipulators in Space Using the Virtual Manipulator Approach", IEEE International Conference on Robotics and Automation, (Raleigh, North Carolina), pp. 579-585.
30. Vanderplaats, G. N., "Numerical Optimization Technologies for Engineering Design", McGraw Hill Book Company, San Francisco, California.
31. Walsh, G., and Sastry, S., 1991, "On Reorienting Linked Rigid Bodies Using Internal Motion", Proc. IEEE Conference on Decision and Control, (Brighton, England), pp. 1190-1195.
32. Yamada, K., and Yoshikawa, S., 1992, "Arm Trajectory Design of a Space Robot", Proc. of the 18th International Symp. on Space Technology and Science, Kagoshima, Japan.

--
INITIAL DISTRIBUTION LIST

- | | | |
|----|--|---|
| 1. | Defense Technical Information Center
Cameron Station
Alexandria, Virginia 22304-6145 | 2 |
| 2. | Library, Code 52
Naval Postgraduate School
Monterey, California 93943-5002 | 2 |
| 3. | Department Chairman, Code ME
Department of Mechanical Engineering
Naval Postgraduate School
Monterey, California 93942-5000 | 2 |
| 4. | Professor R. Mukherjee, Code ME/Mk
Department of Mechanical Engineering
Naval Postgraduate School
Monterey, California 93943-5000 | 3 |
| 5. | Curricular Officer, Code 34
Department of Naval Engineering
Naval Postgraduate School
Monterey, California 93942-5000 | 1 |
| 6. | LT Mary Zurowski
3328 Riverspray Cres.
Mississauga, Ontario
Canada
L4Y-3M4 | 3 |